Visual Models, Procedural Fluency, and Multiple Strategies:

A Cognitive Critique of

Common Core Elementary Mathematics

By Qianruo Shen

Abstract: This paper examines the cognitive and curricular problems of Common Core elementary mathematics. Through analysis of Big Ideas Math and related instructional practices, it argues that, visual representations are employed excessively and without sufficient transition to symbolic reasoning and mathematical abstraction. At the same time, standard algorithms and procedural memory are de-emphasized, while multiple strategies are treated as goals in themselves rather than as means to efficient problem solving.

The paper further argues that the opposition between understanding and memorization reflects a serious cognitive misconception. In practice, the proliferation of visual models and solution methods tends to keep instruction at the level of surface features rather than guiding students toward deeper conceptual structure. As a result, the curriculum becomes diffuse and overloaded, increasing students' burden and extending learning time. Difficulties in fractions among American students reflect these deeper structural and cognitive problems.

The study concludes that elementary mathematics should remain grounded in practicality, clarity, and coherent procedural structure so that most students can achieve solid mastery.

1. Introduction

Over the past half century, the United States has undergone three major waves of K–12 mathematics reform: the "New Math" movement of the 1960s and 1970s, the "Reform Math" movement of the 1980s and 1990s, and, since 2010, the Common Core State Standards for Mathematics.[1] The first two reforms did not achieve their stated goals. The Common Core Standards have now been in place for more than fifteen years, and their effects have likewise been widely questioned.

Before Common Core Math was adopted, NAEP mathematics scores in fourth and eighth grade showed a gradual upward trend nationwide; while afterward, both have declined.[2] Results from PISA and TIMSS have also been disappointing.[3][4]

The Common Core Math Standard has sparked strong backlash in society. Many parents complain that math instruction is confusing, with homework assignments becoming nonsensical; children learn to explain concepts but struggle with calculations. Teachers report that the standards lack practical applicability, classroom rhythms are disrupted, and students' foundational skills have noticeably declined.

Elementary mathematics originates in daily life and is meant to serve practical purposes. Its concepts are concrete, its difficulty moderate. The spread of “math anxiety” to the elementary level in the United States and Canada therefore signals not a failure of students or teachers, but of curriculum design. Over the years, some studies have examined the curriculum and policy dimensions of Common Core.[5][6][7][8] Eric A. Nelson, meanwhile, offered a cognitive-science critique of the standards, particularly their treatment of memorization, procedural knowledge, and automaticity.[9]

“Discovery math” served as a central instructional approach in the first two reform movements. Liping Ma argued that its primary weakness lies in its content and structure—namely, a “multi-strand” organization lacking a clear core.[10] Within this framework, arithmetic was compressed, while topics such as equations and sequences were introduced at the elementary level. Moreover, the strands could be changed with considerable flexibility, resulting in limited coherence and stability.

The Common Core standards have moved away from the “multi-strand” approach, restoring the traditional content and structure of elementary mathematics—an improvement that deserves recognition. However, the influence of discovery math remains significant. Its emphasis on “understanding over memorization” and the so-called inquiry-based instruction have not only persisted but have been amplified and systematized, particularly in the cognitive pathways they promote. This study suggests that such deviations have contributed to the continuing difficulties in elementary mathematics learning, rather than leading to the expected improvements.

This article examines the cognitive flaws of the Common Core Elementary Mathematics Standards through a close analysis of *Big Ideas Math* textbooks.[11] It focuses on arithmetic—numbers, the four operations, and the development of fluency.

2. Improper and Excessive Use of Visual Representations

In *Big Idea Math* textbooks, many dazzling icons and diagrams immediately catch the eye: grids, circles or dots, blocks, 10x10 number charts, number lines, arrays, and more. When handling any concept, they are employed even throughout the entire process.

The extensive use of visual representations is a long-standing feature of Western mathematics education. When used appropriately, such tools can make abstract ideas visible, clarify structure, and help students form mental images. For example, place value is clearly illustrated with base-10 blocks; the number line is essential for introducing zero and negative numbers; and the understanding of fractions and percentages benefits from grids and circles, where part–whole relationships are not easily grasped through symbols alone.

Effective use of visual representations depends on precise verbal explanation and symbolic operations to clarify underlying structure and logic. However, Big Ideas Math places excessive reliance on diagrams while neglecting these supports; as a result, even accurate visuals often fail to achieve their intended effect.

When overused or misapplied, visual methods become even counterproductive. Unfortunately, both discovery-based mathematics and Common Core curricula frequently fall into this latter category.

The following are typical situations of misuse of visual models in Common Core elementary mathematics.

- **Using visual tools for calculation.** Visual representations are intended to support conceptual understanding, not to serve as tools for computation—which is often inefficient, cumbersome, and imprecise—yet such practices have become widespread.

A particularly problematic example is the use of a 10×10 number chart for addition and subtraction through counting on or back—a method no more efficient than simple counting on fingers. Similarly, the use of the number line for addition, subtraction—especially with large numbers—poses practical difficulties, as students cannot locate results with sufficient precision.

The use of arrays to perform multiplication is impractical too. Such tasks can be handled far more efficiently through mastery of the multiplication table.

- **The mismatch between tools and objects.** In some cases, the visual model does not match the structure of the concept it is meant to illustrate.

Number line is used everywhere, but some of its applications are improper. Number line effectively represents addition—moving to the right, and subtraction—moving to the left. However, multiplication is not about directional movement but about structural scaling. The number line fails to capture this essence, applying it in this context reflects a mismatch between tool and object, making a second-order operation remain at the first-order level.

- **Replacement of real-life examples by visual models**

Topics of elementary mathematics are closely related to everyday life. When introducing new concepts, instruction should ideally begin with familiar, real-life examples that engage students and stimulate curiosity. However, *Big Ideas Math* often introduces concepts through visual diagrams, while everyday contexts are underused or deferred.

For example, multiplication and division can be introduced through simple, concrete situations: "A box holds 12 eggs; how many eggs are in 4 boxes?" or "40 students are divided into 5 groups; how many are in each group?" Such intuitive and meaningful questions can strongly stimulate students' thinking, yet they are rarely seen in the textbooks. Fractions, too, are commonly introduced through circles, grids, and number lines, with limited connection to practical contexts. Another example, the distributive property can be illustrated through everyday situations, such as calculating the cost of a shared meal, yet textbooks begin with grid diagrams.

As a result, the visual models often displace the natural starting point of learning—from concrete experience—and weakens the development of conceptual understanding at its earliest stage.

- **Continual use of visual models hinders abstraction.** Visual models are used throughout the learning process, often extending to stages where they are no longer appropriate.

Base-10 blocks, for example, provide a clear representation of place value in early stages. However, when decimals are introduced, continued reliance on these models are both confusing and unnecessary: since such diagrams become increasingly complex and difficult to interpret, meanwhile students have internalized the underlying concept.

Visual representations accompany all four basic operations — addition, subtraction, multiplication and division — often to the end of the process, while symbolic computation is relatively marginalized.

Yet visual models are not the endpoints of learning. Their purpose is to support understanding and guide students toward abstract thinking—not to replace it. Therefore, visual representations should be used with restraint and should give way, in a timely manner, to abstract mathematics. Sustained dependence on visual aids keep students at a low, concrete, and immature level of thinking, ultimately impeding rather than advancing cognitive development.

To summarize, there is broad agreement on the proper role of visual models, across the history of mathematics, findings in cognitive science, and classroom practice. Visual representations are not central; they are to illustrate underlying structures at appropriate stages, not to dominate the learning process. The repeated, indiscriminate, and prolonged use of visual models has become a significant weakness. It reflects a misunderstanding of the nature of mathematics.

3. Devaluation and Weakening of Procedural Rules and Fluency

When visual models are treated as the primary path to understanding, well-established procedural knowledge and skills are regarded as rigid routines that students can't understand, thus are devalued and weakened.

American students' limited mastery of standard column algorithms for the four basic operations provides a clear example of this marginalization.

Long proven to be both reliable and efficient, standard algorithms remain the most powerful tools for numerical computation in elementary mathematics. Their effectiveness rests on the base-10 place-value system and the distributive property: numbers are decomposed into place-value units (ones, tens, hundreds), operated on, and then recombined through systematic carrying and borrowing. In this way, calculations become structured, transparent, and repeatable.

These methods are not inherently difficult. With clear explanation and carefully sequenced examples, students can develop both understanding and mastery. Yet in practice, common errors—such as misalignment of place value or incorrect subtraction procedures—appear frequently in classroom work and assessments.

Such difficulties are often attributed to a "lack of conceptual understanding" or to excessive reliance on rote memorization. In reality, however, the problem often lies in the opposite direction: while visual models occupy substantial instructional space, the establishment of procedural principles is not clear and firm enough, and their practice is far less than needed.

The traditional progression of procedural learning is well established: from principle, to rule formulation, to memorization, to practice, to fluency, and ultimately to automaticity. Once the underlying principles are understood, the crucial step is to establish clear operational rules and reinforce them through practice.

As Eric A. Nelson[9] and Stanford mathematicians R. James Milgram and Ze'ev Wurman[12] have noted, the Common Core standards place insufficient emphasis on automaticity and procedural fluency—foundational elements of mathematical achievement in high-performing systems. Without it, even "understanding" remains slow, effortful, and unreliable.

When basic rules are not firmly established and practiced, the progression from understanding to fluency and automaticity cannot be completed. This weakness at the foundation of arithmetic makes further development of mathematical competence increasingly difficult.

4. Goal Drift: Overemphasis on Multiple Strategies

In Common Core mathematics, multiple strategies are often treated as a hallmark of "understanding," leading elementary mathematics to drift away from its core objectives.

To cross a river, one might wade, swim, take a boat, or build a bridge. As long as the other bank is reached, the goal has been achieved. No one would repeat every possible approach. *Big Ideas Math, h*owever, systematically presents all strategies for every problem, and expects students to learn each one.

While it is valuable for a problem to admit more than one solution, multiple strategies are neither the goal nor a necessity. The primary purpose of mathematics is effective problem solving. Students need to recognize which methods are more fundamental, efficient, and broadly applicable. Precision and economy constitute the true elegance of mathematics.

For example, in addition and subtraction within 100, alongside number charts and number lines, students are introduced to a wide range of named strategies—such as partial sums, regrouping, using doubles, compensation, and others. While some of these may serve as useful mental shortcuts, they do not all require formal and systematic instruction. Not to mention some methods such as "using doubles" are basically useless.

In contrast, with a good number sense of "making a ten", the standard column algorithm can solve the problems efficiently.

A similar issue appears in two-digit by two-digit multiplication. Again *Big Ideas Math* presents a long list of methods: place-value (tens×tens+tens×ones+ones×tens+ones×ones); area models (total area=sum of areas of the 4 small rectangles); distributive property (for example, $14\times23=(10+4)\times(20+3)=10\times20+10\times3+4\times20+4\times3$); and partial products, regrouping, etc.

Yet these are not fundamentally different methods; but variations built on the same underlying structure. Treating them as separate strategies can turn a straightforward task into an unnecessarily complex and time-consuming task. As instructional time is spread across multiple strategies, practice of the standard algorithm is reduced, making it difficult for students to achieve fluency and automaticity.

Fragmented techniques are often presented as "innovations" to demonstrate diversity of methods. This proliferation without clear priority leaves the curriculum diffuse and unfocused, and may hinder rather than deepen understanding.

5. Memorization, Understanding, and Structure

"Understanding" has become a central slogan in Common Core mathematics. The instructional practices discussed above—heavy reliance on visual models, the de-emphasis of procedural rules, and the promotion of multiple strategies—are all intended to serve this goal.

A widely observed pattern in American K–12 mathematics is that, students' difficulties become most apparent from the upper elementary to middle school years. Earlier content often involves basic numerical ideas that students can grasp through everyday experience. However, when instruction reaches topics requiring systematic learning—such as fractions and their operations—many students encounter significant difficulties.

These difficulties are often attributed to a "lack of comprehension" or to "rote memorization." In practice, however, American students typically memorize neither multiplication tables nor basic formulas, which are often provided during examinations. In this sense, memorization is used far less than children's learning capacity would allow.

Framing understanding and memorization as opposites—praising the former while dismissing the latter—reflects a misconception. In reality, they are interdependent: understanding supports memory, and memory provides the basis for deeper understanding. Without memorization and practice, the so-called "understanding" is often vague and unstable.

The pattern of memorizing first and understanding later is not uncommon, especially for children. When complete conceptual clarity is not immediately attainable, rules still need to be learned and practiced, otherwise, students risk staying in a state of partial understanding and stagnation.

Mathematics is fundamentally concerned with concepts, structures, and relationships, and instruction ultimately aims at conceptual clarity and the development of abstract thinking. Learning should proceed from phenomena to underlying structure—that is, from the concrete to the abstract, and from surface features to deeper logic.

Genuine conceptual structures are typically simple and unified; deep understanding is reflected in clarity and economy. However a proliferation of diagrams and methods tend to keep learning at the level of surface features[13], rather than guiding students toward underlying structure. As a result, the curriculum becomes diffuse and overloaded —fragmented, drifting, and lacking clear direction—while also increasing students' burden and extending learning time.

This helps explain why many American students struggle to develop mathematical abstraction at an appropriate stage.

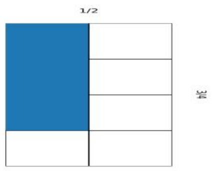
6. Why Fractions Become a Barrier for American Students

An area of weakness for American students lies in fractions and their operations—TIMSS and PISA have repeatedly highlighted this problem—their learning process and outcomes clearly reflect the cognitive issues discussed above.

In *Big Ideas Math*, visual models—rectangles, circles, and number lines—are used extensively, and basic fraction concepts occupy a substantial portion of instructional time. Remarkably, the textbook devotes about 60 pages to addition and subtraction with like denominators—a relatively straightforward topic.

By contrast, operations with unlike denominators—which represent both a central objective and a major source of difficulty—receive comparatively limited and fragmented treatment. The common-denominator method, a standard and efficient approach, is often presented as only one option among multiple strategies rather than as a core procedure to be mastered.

Fraction multiplication and division likewise generate a wide range of errors: multiplying a whole number to a fraction by timing it with both the numerator and the denominator; using cross-multiplication in place of multiplying numerators and denominators separately; and in division, inverting the dividend rather than the divisor.



The figure represent an area model for (1/2)×(3/4)=(1×3)/(2×4), the shaded area includes 1 × 3 small rectangles, whereas the whole figure contains 2 × 4; hence the product is 3/8.

Textbooks employ both tape diagrams (one-dimensional) and area models (two-dimensional) to represent fraction multiplication. While the area model is generally more appropriate, it is often tied to specific examples and lacks clear generalization. It does not adequately explain why numerators and denominators should be multiplied separately, or why dividing by a fraction is equivalent to multiplying by its reciprocal.

Moreover, when finding common denominators, textbooks often use the product of denominators rather than the least common multiple. In multiplication and division, students are not required to simplify fractions during intermediate steps. Systematic instruction in fraction reduction is often delayed, leaving gaps in students' procedural fluency.

American students are generally able to recognize fractions in visual models and handle simple comparisons. However, when the operations require rule-based procedure without visual support, performance declines sharply, since students do not know how to proceed.

A 2022 study illustrates this problem.[14] Students were asked to compare 2/3, 3/4, and 3/8. Researchers identified as many as 12 different strategies, including visual, symbolic, and verbal approaches. Among 214 students, roughly one quarter gave no answer, and nearly half relied on visual methods—yet only 28% reached the correct conclusion. By contrast, about 2/3 of those using a common-denominator method answered correctly.

The key issue is not simply variation in performance across strategies. Rather, the study interprets the solution as encouraging students to learn multiple approaches, without addressing the limitations of certain methods. In doing so, it risks reinforcing the very confusion that the results reveal, rather than strengthening core procedural mastery.

7. Concluding Reflections

Elementary mathematics serves all children. Its primary task is to equip students with essential skills for life—an aim that curriculum standards and textbooks must consistently uphold. Interest and practicality are central and mutually reinforcing; when properly maintained, they enable most students to achieve solid mastery. Conceptual understanding and thinking abilities emerge naturally from this process; when mathematics is detached from practical experience, abstraction loses its grounding.

Within K–12 education, elementary mathematics has been especially susceptible to reform and extensive redesign. Yet as the foundation of both mathematics and science learning, its weakening carries long-term consequences. Insufficient fluency and unstable conceptual structure at this stage translate into later difficulties in algebra, trigonometry, geometry, and the sciences. Over time, repeated struggle shapes' confidence and contribute to the perception of STEM fields as difficult and inaccessible.

The United States has long been a leader in innovation. Yet education is not primarily an arena for rapid innovation, but a social function grounded in the transmission of knowledge; changes within it therefore require clear limits and careful validation. Reforms that alter foundational knowledge structures and cognitive pathways demand particular caution—an essential principle that has been overlooked in contemporary educational discourse.

As Frederick M. Hess has observed,[5] the Common Core was implemented on a nationwide scale with limited prior testing or built-in safeguards. The costs of such large-scale reforms extend beyond ineffective outcomes, affecting the cognitive development of an entire generation, and the scientific and economic competence of the nation.

Restoring stability and quality in education depends on reaffirming this principle and pursuing steady, carefully considered improvements over time.

May 2026

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