How Has Jo Boaler Been Transforming U.S. K-12 Math ?

Section I: Articles and Papers Questioning Jo Boaler's Studies

1. https://educhatter.wordpress.com/2018/09/11/multiplication-and-math-practice-do-jo-boalers-math-theories-hold-water/
   Multiplication and Math Practice: Do Jo Boaler's Math Theories Hold Water?

2. http://www.danielwillingham.com/daniel-willingham-science-and-education-blog/march-13th-2019
   Objections to Jo Boaler's Take on Neuroscience and Math Education

3. http://www.sanctuarymedia.com/paperlessmojo/jo-boaler-is-half-wrong-the-many-fallacies-of-mathematical-mindsets/
   Jo Boaler is (Half) Wrong — The Many Myths in Mathematical Mindsets

4. https://educationrealist.wordpress.com/2013/01/16/jo-boalers-railside-study-the-schools-identified-kind-of/
   Jo Boaler's Railside Study: The Schools, Identified. (Kind of.)

5. https://thequirkyteacher.wordpress.com/2017/08/11/jo-boaler-is-wrong-about-maths-facts-and-timed-tests/?blogsub=spammed#blog_subscription-2
   Jo Boaler is wrong about maths facts and timed tests

6. https://nonpartisaneducation.org/Review/Essays/v8n4.htm
   A Response to Some of the Points of: When Academic Disagreement Becomes Harassment and Persecution

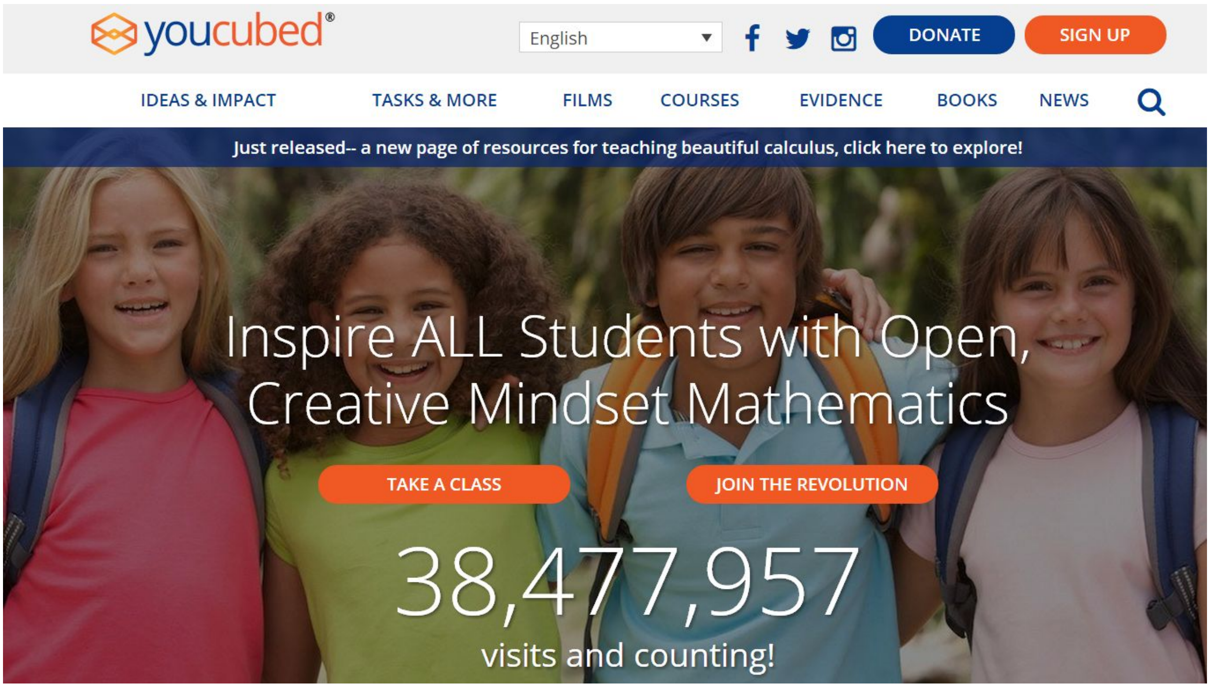
7. https://nonpartisaneducation.org/Review/Articles/v8n1.pdf
   A Close Examination of Jo Boaler's Railside Report

8. http://www.qedcat.com/misc/141.html
   Boaler? Boaler?

Boaler’s Youcubed.org tallied more than 38 millions of visits. But her courses had dismal enrollment at Stanford!

Boaler's CME 10A course on "big ideas of calculus" offered to a small group of incoming students in the summer at Stanford will meet the same fate as Boaler's earlier CME 10 course "How to learn math" (it no longer exists, and had dismal enrollment in its 2nd = final offering).

The Math department at Stanford continues to teach single-variable calculus with the same traditional content as always (e.g., see the course schedule and homework on the Math 21 webpage from Fall 2019 via Google search) and bans the use of calculators on exams its calculus courses (see the "Exam overview" section on the main Math 21 page, for example). A summer course that does not provide the traditional skills necessary for success in learning actual calculus is going to fizzle out just like CME 10 did.



Jo Boaler claims that *College Preparatory Mathematics (CPM)* had significantly improved students' math performance in her Railside Report. *CPM* is currently in use at the San Francisco Unified School District, where Boaler practices her beliefs in detracking, mixed ability teaching, group work, banning algebra in middle schools, and formative assessments.

CPM is one of the ten textbooks which have ignited a protest by 220 leading mathematicians and scientists, including 7 Nobel Laureates and Fields Medalists, who signed an open letter published in the Washington Post on November 19, 1999.

AN OPEN LETTER TO UNITED STATES SECRETARY OF EDUCATION, RICHARD RILEY

Dear Secretary Riley:

In early October of 1999, the United States Department of Education endorsed ten K-12 mathematics programs by describing them as "*exemplary*" or "*promising*." There are five programs in each category. The "*exemplary*" programs announced by the Department of Education are:

Cognitive Tutor Algebra
College Preparatory Mathematics (CPM)
Connected Mathematics Program (CMP)
Core-Plus Mathematics Project
Interactive Mathematics Program (IMP)

The "*promising*" programs are:

Everyday Mathematics
MathLand
Middle-school Mathematics through Applications Project (MMAP)
Number Power
The University of Chicago School Mathematics Project (UCSMP)

These mathematics programs are listed and described on the government web site: http://www.enc.org/ed/exemplary/

The Expert Panel that made the final decisions did not include active research mathematicians. Expert Panel members originally included former NSF Assistant Director, Luther Williams, and former President of the National Council of Teachers of Mathematics, Jack Price. A list of current Expert Panel members is given at: http://www.ed.gov/offices/OERI/ORAD/KAD/expert_panel/mathmemb.html

It is not likely that the mainstream views of practicing mathematicians and scientists were shared by those who designed the criteria for selection of "*exemplary*" and "*promising*" mathematics curricula. For example, the strong views about arithmetic algorithms expressed by one of the Expert Panel members, Steven Leinwand, are not widely held within the mathematics and scientific communities. In an article entitled, "*It's Time To Abandon Computational Algorithms*," published February 9, 1994, in Education Week on the Web, he wrote:

> "*It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive. In fact, it's time to acknowledge that continuing to teach these skills to our students is not only unnecessary, but counterproductive and downright dangerous.*" (http://www.edweek.org/ew/1994/20lein.h13)

In sharp contrast, a committee of the American Mathematical Society (AMS), formed for the purpose of representing the views of the AMS to the National Council of Teachers of Mathematics, published a report which stressed the mathematical significance of the arithmetic algorithms, as well as addressing other mathematical issues. This report, published in the February 1998 issue of the Notices of the American Mathematical Society, includes the statement:

> "*We would like to emphasize that the standard algorithms of arithmetic are more than just 'ways to get the answer' -- that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials.*"

Source: http://www.csun.edu/~vcmth00m/riley.html

David Klein, A Brief History of American K-12 Mathematics Education in the 20th Century

David Klein, A Quarter Century of US 'Math Wars' and Political Partisanship

For more information on the dumbing-down of US K-12 math by progressive educators, please visit http://www.studibee.org/?page_id=348 .

<u>Section II: List of Journal Papers by Jo Boaler:</u> Equity is the main focus of Jo Boaler's journal papers.

1. Achieving Elusive Teacher Change through Challenging Myths about Learning: A Blended Approach
2. Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives
3. Mathematics and science inequalities in the United Kingdom: when elitism, sexism and culture collide
4. Changing students' lives through the de-tracking of urban mathematics classrooms
5. Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach
6. When politics took the place of inquiry: A response to the National Mathematics Advisory Panel's review of instructional practices
7. Creating mathematical futures through an equitable teaching approach: The case of Railside School Teachers College Record Boaler, J., Staples, M.2008; 110 (3): 608-645
8. Urban success: A multidimensional mathematics approach with equitable outcomes
9. How a detracked mathematics approach promoted respect, responsibility, and high achievement
10. When learning no longer matters: Standardized testing and the creation of inequality
11. Learning from teaching: Exploring the relationship between reform curriculum and equity
12. The development of disciplinary relationships: Knowledge, practice and identity in mathematics classrooms
13. Paying the price for ' sugar and spice ': Shifting the analytical lens in equity research
14. Learning from teaching: Exploring the relationship between reform curriculum and equity
15. Exploring the nature of mathematical activity: Using theory, research and working hypotheses' to broaden conceptions of mathematics knowing
16. Mathematical modeling and new theories of learning Teaching Mathematics and Its Applications
17. Mathematics from another world: Traditional communities and the alienation of learners
18. Students' experiences of ability grouping-disaffection, polarization and the construction of failure
19. Participation, knowledge and beliefs: A community perspective on mathematics learning
20. Open and closed mathematics: Student experiences and understandings
21. Mathematical equity—underachieving boys or sacrificial girls?
22. Alternative approaches to teaching, learning and assessing mathematics
23. Reclaiming school mathematics: The girls fight back
24. Equity, empowerment and different ways of knowing
25. When even the winners are losers: Evaluating the experiences of top set' students
26. Setting, social class and survival of the quickest
27. Learning to lose in the mathematics classroom: A critique of traditional schooling practices in the UK
28. The Role of Contexts in the Mathematics Classroom: Do they Make Mathematics More ' Real '?
29. Encouraging the transfer of 'school' mathematics to the 'real world' through the integration of process and content, context and culture
30. The 'psychological prisons' from which they never escaped: The role of ability grouping in reproducing social class inequalities
31. Students' Experiences of Ability Grouping - disaffection, polarization and the construction of

failure

Ze’ev Wurman on why progressive educators sacrifice students’ academic achievements on the altar of equality of outcomes:

All the dumbing-down agenda — including the latest campaign on removing algebra 2, is not because all the leading educationists are stupid. Many are, but many are not. The only way I can, after all those many years, to explain to myself what is going on is that all those educationists -- I don't even want to call them educrats, as this is too complimentary to them -- don't really care about education, and they don't actually believe in the importance of education. The reliance on Rousseau and educational romanticism is, in my opinion, largely fake. What they really are after is equality -- equality of outcomes.

They see all the data that educated people get higher salaries and are more successful and, like in the Polynesian Cargo Cult (https://en.wikipedia.org/wiki/Cargo_cult), they believe that if people of every color of skin will have equal probability to show a diploma, all of them will deserve equal jobs and equal salaries. They don't believe that, or they don't care whether there is actual talent, ability, or competence hiding behind those diplomas.

That's their Utopia, and on its altar they don't really care to sacrifice talent or merit. Somehow they believe that the world progresses on its own by the armies of mediocrity, so they role is only to try to arrange for as perfect equality as possible.

This may sound bleak, but I see no other explanation to what we see. The awful results hit those educationists in the face for decades, yet they never correct their course. Why, if not because they have different goals from you and me? After all, not all of them are stupid or charlatans.

The American educational system still has pockets of excellence, but they are being extinguished one by one. Consider the current effort to do away with the specialized high schools in NYC. So while pockets of excellence still exist, the average for the overwhelming majority is mediocre. The only saving grace of our education is the ability to have private and charter schools, and some of them are pretty good.

In any case, this is my (dark) current view of our education. Perhaps I am too pessimistic. Time will tell.

For more information on understanding progressive educators’ obsession with equality of outcomes and low academic standards, please read:

“Why Have American Schools Failed in Closing the Achievement Gap? A Case Study of the Palo Alto School District.”

Section III. Collections of Jo Boaler's Viewpoints

Sources: all excerpts are from the internet; copy a sentence or paragraph into the Google search box and you will find its original source.

1. Jo Boaler's Immense Influence on US and International K-12 Math Education

Professor Boaler began teaching at Haverstock School in North London before moving into academia with a master's and PhD in mathematics education at King's College London. Her supervisor was Paul Black, renowned as the co-author with Dylan Wiliam of Inside the Black Box, the influential book on Assessment for Learning.

Jo Boaler, professor of mathematics education at Stanford University, has big ambitions – she's on a mission to transform maths education.

She ends each email to the 60,000 people signed up to her www.youcubed.org website with "Vive la révolution" and it's not just a rhetorical flourish.

"It is revolutionary to change maths in this way," she says. "It is going to take a revolution for people to think [differently] about what the subject is."

When Jo Boaler arrives in Massachusetts to teach her new approach to math instruction, she's received as something close to a rock star.

Dan Schwartz, dean of the Graduate School of Education, calls Boaler the public face of K-12 mathematics reform. Although many colleagues across the country share similar views about what needs to be done, he says, her influence with teachers and parents is unmatched. "She is the great communicator."

2. How Traditional Math is Wrong

She begins by pointing out that many children think that maths is a subject in which all the answers are right or wrong. And they are wrong. The misapprehension is understandable, though. Professor Boaler points out that the version of maths taught in many school classrooms is "narrow and impoverished", focused on coming up with the right answers quickly.

Boaler — who teaches at Stanford — travels the country with a message of hope for teachers. There are obvious flaws, she says, in a system that uses stressful tests to decide who's got a brain for math and counts on rote memorization to build mathematical curiosity. With her talks, her research, and a website full of videos about mathematics, her mission is not to build memories — but mindsets.
I think there is a lot of pressure on teachers to be able to cover all of the standards within the short time frame they have. There is a lot of pressure to be successful. I think we need to take a step back from that and think of mathematics in a little bit of a different lens and find the joy in mathematics that's beautiful and interesting and engaging. Oftentimes teachers don't have the ability to spend time on that, because they're so stressed and focused on high-stakes testing and other state frameworks.

By the time students moved on to algebra, where they were using the same concepts but with letters as well as numbers, they were flummoxed. Years of study hadn't stuck. "To them it was just a whole new set of procedures," Baggett says. "They were not making sense of the math; they were just following procedures to get the right answers. They were basically computers when you punch in a number sentence and an answer comes out."

Traditional educators believe that some students do not have the brains to be able to work on complex mathematics, but it is working on complex mathematics that enables brain connections to develop. Students can grasp high-level ideas but they will not develop the brain connections that allow them to do so if they are given low-level work and negative messages about their own potential (Boaler & Foster 2014).

3. How Reform Math is Right

"Real math is visual, exciting, creative, and accessible" — Dr. Jo Boaler
Stanford University's Dr. Jo Boaler demonstrates in her interactive workshops, the need and methodology to open up math to teachers, parents and students. She believes that math was never meant to be taught by boring, procedural, and rote-learning calculations that are then timed and corrected.

Traditionally, math education provided access to a small number of students who were perceived as "good at math." The new approach provides access to all students to learn the mathematics they need for day-to-day life and to increase the number of students that enter fields in science, technology, engineering, and math (STEM).

In her book Mathematical Mindsets, she argues that there should be less testing in maths, less worrying about failure, more use of visual representations and "manipulatives" (items such as blocks and cubes that students can handle) and more emphasis on group work to solve complex problems.

For instance, Boaler is an advocate of "number talks," in which students work on a problem — say, 5 x 18 — then discuss the different ways each approached it. The idea is that by discussing, comparing and visualizing their differing approaches, students build their own sense of context, connection and numeracy.

As I work with schools and districts, encouraging mathematics teaching that promotes growth rather than fixed mindsets (www.youcubed.org), a critical requirement is that teachers offer mathematics as a learning subject, not a performance subject. Most students asked what they think their role is in math classrooms say it is to answer questions correctly. They don't think they are in math classrooms to appreciate the beauty of mathematics, to explore the rich set of connections that make up the subject, or even to learn about the applicability of the subject; they think they are in math classrooms to perform. Students from kindergarten upward realize that math is different from other subjects: learning gives way to answering questions and taking tests—performing.

For students to see mathematics as a subject of learning, not performing, they need tasks and questions in math class that have space to learn built in. When students spend all their time in math class answering discrete questions to which the answers are either right or wrong, it is very difficult

to develop a growth mindset or to believe that mathematics is about growth and learning.

If a mathematics question or task does not have space within it to think, learn, and discuss, its potential as a learning task is limited. Tasks that are particularly valuable are those that have a low floor and a high ceiling—that is, anyone can access them, but they can be taken to very high levels.

4. Traditional Math Hinders Equity

“Some people revel in the inaccessibility of mathematics as it’s currently taught, especially if their own children are succeeding, because they want to keep clear a societal advantage… Mathematics has the greatest and most indefensible differences in achievement and participation for students of different ethnicities, genders, and socioeconomic income levels for any subject taught in the United States.”
And this disparity really matters, because math education is such a powerful predictor of future success.

Most universities require at least three years of high school math for college entrance, determining the opportunities available for the rest of student’s lives.

Earnings can also be predicted by amount, type and level of math course taken in high school. In 2004, Rose and Betts found that the more math classes students take in high school, especially in algebra and geometry, the higher their earnings were ten years later. Moreover, taking advanced math courses predicted an increase in salary as high as 19.50% ten years later.

According to her research, Boaler says there are some extremely successful strategies for producing more equitable math outcomes. She can frequently be heard calling for a “math revolution” to implement these approaches and narrow the achievement gap.

Between 2010 and 2014 more than sixty percent of black students failed their end-of-course (EOC) algebra exams (compared with less than 20 percent of white students). Performance on the EOC geometry test was only slightly better. Even with the new “Smarter Balanced” exams, which are based on recently implemented Common Core Standard, this “achievement gap” doesn’t seem to have narrowed. It exists for other demographic groups like Latinos, American Indians and migrants as well. And it’s not just SPS. The achievement gap is replicated in school districts across the state.

So what’s wrong?
At the classroom level, it has to do with inequitable teaching practices, says a revolutionary math education researchers at Stanford University.
Jo Boaler, author of a new book Mathematical Mindsets, envisions an academic world where everyone can learn math equitably regardless of their skin color, gender, sexual orientation or income.

Jo Boaler, Professor of Mathematics Education, criticizes approaches to math education in the US as inequitable.

Give girls and students of color additional encouragement to learn math and science:
Due to the severely damaging stereotypes that they’ll underperform at math, girls and students of color need more encouragement than other students.

"Both girls and students of color — particularly underrepresented minorities — need thoughtful and positive messages to be given to them, about their valued place in mathematics," Boaler wrote. She gives a few examples how positive messages can be delivered to girls and students of color: Highlighting the achievements of women and underrepresented minorities in mathematics and STEM. Structuring classrooms so students are asked to become an expert on a subject and share their findings with other students. Increasing diversity of teachers to present role models to students of color.

"My main point is that it may not be enough, as a math teacher, to treat students equally in the pursuit of equity," she wrote. "Some students face additional barriers and disadvantages, and we must work to address those quite deliberately if we are to achieve a more equitable society."

What I get from Jo Boaler's work is that we need to rethink how we teach mathematics. The methods that worked for mathematics teachers are not the methods we need to be using for everyone. The defence "The old ways worked for me" is not defensible in terms of inclusion and equity.

Some high-achieving mathematicians who sat through traditional studies and were able to take mathematics to a high level think, "That worked for me, so it should work for everybody." There are also political undercurrents to keeping the system as it is now - some people don't want teaching to be more equitable. They do not want to open the doors to everyone.

We know that the way math is taught now appeals to boys more than girls. Particularly the traditional teaching and rules - boys can turn that into competition. Girls, more than boys, seek to understand deeply, to make connections between math and the real world. They seek a more connected approach to the subject. And teaching in more connected ways enables both boys and girls to do better. In the absence of that, it is instead more procedural. When we take a better teaching approach, everyone is more successful.

Boaler has long promoted mathematics education reform and equitable classrooms, aiming to elevate both achievement and enjoyment, particularly among girls and students of color.

The Gatekeeper title refers to the way that sub-par math scores often restrict access to higher education and good jobs. Boaler, she says, is a major part of the movement to change that.

There's a widespread assumption is that "social-cultural context" is to blame for the math achievement gap. But there's still shockingly little academic research on how context actually affects engagement in students of color (here's the one example I found, based on a survey of studies conducted by Tyrone Howard).
Regardless, Boaler's six strategies are all designed to mitigate those contexts and challenge education inequity.

5. Traditional Math is Based on a Fixed Mindset; Reform Math is Based on a Growth Mindset

At root, Boaler is trying to tap the power of a growth mindset, the concept made famous by Stanford psychologist Carol Dweck, who is a Youcubed adviser and wrote the foreword to Boaler's most recent book, *Mathematical Mindsets.* "Boaler is one of those rare and remarkable educators who not only know the secret of great teaching but also know how to give that gift to others,"

Dweck wrote.

Dweck's research shows that people who see intelligence as something that can be developed handle setbacks better than those who see intelligence as a fixed quality. Someone with a growth mindset is more likely to put in effort to improve because she believes it is possible, while a person with a fixed mindset more readily interprets failure as revelation of uncontestable limits. "I'm not a math person" reflects a fixed belief.

Mathematical Mindsets has a foreword from Professor Boaler's Stanford colleague Carol Dweck, the psychologist who has inspired many teachers with her work on the importance of a "growth mindset" and encouraging children to believe that talent is not something you are born with but something you can work towards.
Communicating that change to students in the classroom is both about what you say and don't say: Don't tell students they are "smart" when they get perfect score. Parents should never say they are "bad in math" in front of their children — especially girls. Both things just reinforce that "fixed mindset."

As in most STEM fields, in math education there tends to be a belief that aptitude for learning math is something genetically owned and not modifiable. Boaler describes this "fixed mindset" as "the heart of inequity problems in Mathematics."

This held across all 30 subjects they considered (see also [8] this volume). Mathematics professors held the most fixed views of any STEM field. Boaler [9] has proposed that the idea that only some people can be successful in mathematics is at the root of the widespread mathematics anxiety that pervades the US and elsewhere. Mathematics anxiety has been shown to reduce performance [10], and to be passed on from teachers and parents to students.

6. Reform Math Celebrates Mistakes

"We need to free our young people from the crippling idea that they must not fail, that they cannot mess up, that only some students can be good at maths and that success should be easy and not involve effort," her book concludes. She says that at its core, maths is about not simply getting a string of right answers but rather reasoning – that is, how you get to those answers.

Jason Moser and his colleagues actually found from MRI scans that your brain grows when you make a mistake in maths. Fantastic. When you make a mistake, synapses fire in the brain. And in fact, in their MRI scans they found that when people made a mistake synapses fired. When they got work correct, less synapses fired. So making mistakes is really good. And we want students to know this.

So it's really important that we change the messages kids get in classrooms. We know that anybody can grow their brain, and brains are so plastic to learn any level of maths. We have to get this out to kids. They have to know that mistakes are really good.
"Every time a student makes a mistake in math, they grow a synapse." "..mistakes cause your brain to spark and grow." "Synapses fire when learning happens".

Then we tell them that mistakes actually do amazing things in your brain and when you make a mistake, synapses fire in the brain — more than if you get it right. There's brain strengthening,

brain growth — all sorts of good things are happening when you make mistakes in the brain. So mistakes are very important.

That goes against so much of what maybe the parents of students today have learned — that getting everything right is how you know you're learning.

I think what people believe about math is pretty wrong. And everything that goes on in math class is pretty wrong. We know that when you're making mistakes — when you're challenged — those are the best times of brain growth.

And then we also know the speed and time pressure is damaging for the brain. So when we give kids timed tests, it shuts down the part of the brain that we need to use when they are taking timed tests.

Research has recently shown something stunning—when students make a mistake in math, their brain grows, synapses fire, and connections are made; when they do the work correctly, there is no brain growth (Moser et al. 2011). This finding suggests that we want students to make mistakes in math class and that students should not view mistakes as learning failures but as learning achievements (Boaler 2013a). Students do not, as many assume, need to revisit a mistake and correct it to experience brain growth, although that is always helpful; brain growth comes from the experience of struggle. When students struggle with mathematics, their brains grow; being outside their comfort zone is an extremely important place to be.

Mathematics classrooms throughout the U.S. are often set up to make students feel good by giving them lots of questions they can answer. Teachers believe that mistakes and struggle are unproductive and try to shelter students from them. This culture needs to change.
While I was sitting in an elementary classroom in Shanghai recently, the principal leaned over to tell me that the teacher was calling on students who had made mistakes to share with the whole class so that they could all learn. The students seemed pleased to be given the opportunity to share their incorrect thinking.

That mistakes are valuable and she loves them. Failure and struggles are the most important part of math and learning, and they don't mean her students can't do math.

7. Timed, Closed Tests Should be Replaced with Open-ended Tasks

Instead of classrooms filled with short questions students are intended to get right or wrong, mathematics classrooms need to be filled with open-ended tasks that include space for learning as well as space for struggle and growth (www.youcubed.org). For mathematics to become a learning subject with room for mistakes and growth, teachers need to make students feel good about mistakes and comfortable with struggle. When I taught a recent online class and shared the mistakes research with forward-thinking mathematics teachers, they came up with a range of ways for getting students to value mistakes.

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so is frustrating for students. Boaler [9] has highlighted the importance of changing mathematics questions so they can be consistent with growth mindset ideas. When questions are narrow and closed, i.e., with one correct answer, students are likely to become frustrated if they do not know how to succeed. Over time, they come to see mathematics as a fixed and closed subject. When questions are opened to become low floor and high ceiling, with many entry points and where many different ways of working are valued, students are more likely to experience success and to see mathematics as an open and growth subject.

PISA tests are definitely better because they rely on problem solving. It's not one of the rote, multiple-choice, procedural tests that go on in the United States all the time. I have an article about the PISA assessments in the current edition of Scientific American Mind magazine, and I worked with the PISA analyst. And one thing that the last PISA did was as well as test kids mathematically, they also assessed their approach to learning.

Teachers always know how well kids are doing, so you really don't need to test them. You really easily have teachers write down what kids know and can do. The kids themselves can also self-assess and tell if things are strong or not. They do that with extreme reliability. You can ask kids make a project, if you want, that tells us about what they know and can do.

And most tests used do not assess what's important anymore. They might assess whether you are computationally fast — but that's the one thing computers do and we don't need humans for.

That's really interesting, because we get our data on how students do in math class, or in classes in general, from timed tests. How do you suggest educators measure their progress instead?

Teachers always know how well kids are doing, so you really don't need to test them. You really easily have teachers write down what kids know and can do. The kids themselves can also self-assess and tell if things are strong or not. They do that with extreme reliability. You can ask kids make a project, if you want, that tells us about what they know and can do.

And most tests used do not assess what's important anymore. They might assess whether you are computationally fast — but that's the one thing computers do and we don't need humans for.

Brain research has elucidated another practice that keeps many children from succeeding in math. Most mathematics classrooms in the U.S. equate skill with speed, valuing fast recall and testing even the youngest children against the clock. But studies show that kids manipulate math facts in their working memory—an area of the brain that can go off-line when they experience stress. Timed tests impair working memory in students of all backgrounds and achievement levels, and they contribute to math anxiety, especially among girls. By some estimates, as many as a third of all students, starting as young as age five, suffer from math anxiety.

Most would agree that we want students to feel like their minds can “run wild” with ideas. But so many students experience the opposite feeling. Our grading and testing practices are largely responsible. It’s bad enough when students receive grades at the end of each unit or course that tell them how capable they are, but technological advances like digital grade portals have meant that students can see where their grades stand, and when they change, every minute of every day.

This has amplified the performance pressure on students. Research has shown that students only have to think they're being graded for their achievement to go down. Math teachers who replace grading with constructive written comments increase student's learning.

As students realize they cannot perform well on timed tests they start to develop anxiety and their mathematical confidence erodes. The blocking of the working memory and associated anxiety particularly occurs among higher achieving students and girls. Conservative estimates suggest that at least a third of students experience
extreme stress around timed tests, and these are not the students who are of a particular achievement group, or economic background. When we put students through this anxiety provoking experience we lose students from mathematics.

Math anxiety has now been recorded in students as young as 5 years old (Ramirez, et al, 2013) and timed tests are a major cause of this debilitating, often life-long condition. But there is a second equally important reason that timed tests should not be used – they prompt many students to turn away from mathematics. In my classes at Stanford University, I experience many math traumatized undergraduates, even though they are among the highest achieving students in the country. When I ask them what has happened to lead to their math aversion many of the students talk about timed tests in second or third grade as a major turning point for them when they decided that math was not for them. Some of the students, especially women, talk about the need to understand deeply, which is a very worthwhile goal, and being made to feel that deep understanding was not valued or offered when timed tests became a part of math class.

I love math, but I know that I'm unusual. Math anxiety is a rampant problem across the country. Researchers now know that when people with math anxiety encounter numbers, a fear center in the brain lights up — the same fear center that lights up when people see snakes or spiders. Anxiety is not limited to low-achieving students. Many of the undergraduates I teach at Stanford University, some of the most successful students in the nation, are math traumatized. In recent interviews, students have told me that learning math in school was like being on a "hamster wheel" — they felt like they were running and running, without reaching any meaningful destination. A seventh grader told me that math learning was like prison, because his mind felt "locked up."

The problem is the performance culture in our schools, more present in math than in any other subject. Students believe that the purpose of math class is to demonstrate that they can quickly find the answers. An undergraduate recently told me that when she writes down her ideas, even when working alone, she expects someone to judge her. She's unable to think freely because she's afraid of writing something that isn't "smart enough." This kind of thought paralysis is the direct result of our emphasis on performance, which we urgently need to dial back. My research on math learners suggests that when students think they're in class to learn — to explore ideas and think freely — they understand more and achieve at higher levels than when they think the point is to get questions right.

Until we change the way we teach math to emphasize learning and exploration, rather than performance, we'll continue to produce students who describe their math experience as a hamster wheel, or worse, a prison. We'll continue to produce anxious students who experience fear when they see numbers. The performance culture of mathematics has destroyed a vibrant, essential subject for so many people. As schools have worked to encourage a few speedy calculators, they've neglected to teach the kind of creative, quantitative thinking that can open new worlds. If we

encourage new generations of students who love learning and love math, we'll raise up kids who are prepared to take their place in society as free, empowered thinkers.

One area in desperate need of examination is the way we teach mathematics. Many Americans suffer from misconceptions about math. They think people are either born with a "math brain" or not — an idea that has been disproven — and that mathematics is all numbers, procedures and speedy thinking. In reality, mathematicians spend most of their working lives thinking slowly and deeply, investigating complex patterns in multiple dimensions. We sacrifice many people — women and students of color, in particular — at the altar of these myths about math.

When students struggle in speed-driven math classes, they often believe the problem lies within themselves, not realizing that fast-paced lecturing is a faulty teaching method. The students most likely to internalize the problem are women and students of color. This is one of the main reasons that these students choose not to go forward in mathematics and other STEM subjects, and likely why a study found that in 2011, 74% of the STEM workforce was male and 71% was white.

Women are just as capable as men of working at high speed, of course, but I've found in my own research that they are more likely to reject subjects that do not give access to deep understanding. The deep understanding that women seek, and are often denied, is exactly what we need to encourage in students of mathematics. I have taught many deep, slow thinkers in mathematics classes over the years. Often, but not always, they are women, and many decide they cannot succeed in mathematics. But when the message about mathematics has changed to emphasize slower, deeper processing, I've seen many of these women go on to excel in STEM careers.

We're a math-traumatized people, Jo Boaler says (although she uses the British locution "maths-traumatized"). It's a belief she sees confirmed in everything from students crying over long division to MRIs that reveal young brains reacting to numbers as if they were snakes or spiders. And it's something she hears just as clearly in the resignation of that common refrain: "I am not a math person."

For Boaler, the test — with its focus on speed, volume and performance — is a big part of why math crushes spirits like no other subject. To her, it represents shallow learning with debilitating consequences. Students who work slowly are often left convinced of their own inability, although they may be the deeper kind of thinkers who make the best mathematicians. And even those who calculate speedily — not a skill Boaler thinks is particularly valuable in the digital age — may end up shrugging off math as a high-pressure hamster wheel.

As a researcher, teacher and evangelist, Boaler is a leading voice for a wholly different pedagogy where speed is out, depth is in, and the journey to an answer can be as important as the destination. It's an approach where sense-making matters more than memorization and retaining "math facts" matters less than understanding how such facts interconnect.

> But maths classrooms have to change in a lot of ways. It’s not just about changing messages for kids. We have to fundamentally change what happens in classrooms. And we want kids to have a growth mindset, to believe that they can grow, and learn anything. But it’s very difficult to have a growth mindset in maths. If you’re constantly given short, closed questions that you get right or wrong, those questions themselves transmit fixed messages about math, that you can do it or you can’t. So we have to open up maths questions so that there’s space inside them for learning.

Brain research has elucidated another practice that keeps many children from succeeding in math. Most mathematics classrooms in the U.S. equate skill with speed, valuing fast recall and testing even the youngest children against the clock. But studies show that kids manipulate math facts in their working memory—an area of the brain that can go off-line when they experience stress. Timed tests impair working memory in students of all backgrounds and achievement levels, and they contribute to math anxiety, especially among girls. By some estimates, as many as a third of all students, starting as young as age five, suffer from math anxiety.

A number of leading mathematicians, such as Conrad Wolfram and Steven Strogatz, have argued strongly that math is misrepresented in most classrooms. Too many slow, deep math thinkers are turned away from the subject early on by timed tests and procedural teaching. But if American classrooms begin to present the subject as one of open, visual, creative inquiry, accompanied by growth-mindset messages, more students will engage with math's real beauty. PISA scores would rise, and, more important, our society could better tap the unlimited mathematical potential of our children.

Math Anxiety/Trauma: This is a widespread phenomenon. Many adults and teachers have had bad experiences with the subject of math. Math anxiety is created from painful memories of timed tests, monotony, abstract ideas, endless wrong answers, and the absence of relevance. Most of us have heard someone say, “I am just not a math person.” Boaler is aghast that this deplorable comment is widely accepted as normal and has set out to refute it. Math, in her view, should not cause students to wonder why they are learning it. Math is a dynamic, exciting and relevant subject. She challenges the focus on speed and correct answers in math. She offers failure and mistakes as beautiful teachers. Boaler says she meets adult after adult who look back and are able to identify the message or experience that shaped their negative math self-perception, often tearfully.

There is no ‘math’ wall to hit. Everyone’s brain is capable of learning advanced math with good teaching and powerful mindsets. The truth is that the more you struggle with a problem, (as weight-lifters increase weights to get stronger), the more your brain will grow and learn. Boaler would like to see math change from a performance orientation to a learning orientation.

Boaler dispelled the idea that math needs to be speedy; she says math, as a subject, needs time and space for learning.
Other changes need to happen as well. Mathematics teachers need to stop frequent, timed testing; replace grades with diagnostic feedback (Black et al. 2002; Boaler & Foster 2014); and deemphasize speed, so that the students who think slowly and deeply are not led to believe they are not capable (Boaler 2014).

What is the purpose of continuing the CLA testing? It seems to be perpetuating the fixed

mindset—timed rigid testing.
The evaluation of students' learning is based on the work that they produce, including non-written, participatory evidence. There are several ways to gather these evaluations. Teachers can closely observe their students as they work and take notes. Students can produce an individual product based on their small group work. Most evaluations of students are meant to help the teacher determine where the student is on a learning continuum, so that the teacher can plan the next steps to take in their instruction. There is a large body of research that says evaluative feedback does not enable learning and in fact often causes the student to stop learning. Feedback that is based on next steps for improvement has proved far more effective. With this in mind, student evaluation in the form of grades should be limited, while feedback in the form of next steps should be prolific.

In the elementary grades, we have rewritten the Standards Based Report Cards to reflect the new standards. Students will be evaluated over the course of the academic on the new standards using a '1-4' scale indicating their progress towards mastery. Rubrics for each grade are being developed as we speak and will be available next year.

The district Common Learning Assessments (CLAs) are a vehicle to gather data about student learning at three designated points during the academic year and use the information as a formative assessment to guide instruction. Embedding the constructed response and performance assessment questions from the core curriculum Milestone Tasks in the CLA reduces the amount of testing for our students at all grade levels. CLAs should not be viewed as a hard and fast grade of students' progress.

When students engage in interesting, challenging mathematics, they see the difference between deeply exploring math and following procedures outlined in traditional textbooks. With a curriculum that inspires a growth mindset and teacher training to support this understanding, students will have opportunities to access deep mathematics learning.

They found a unique balance that is now seen as a national model. They decided to challenge students earlier with depth and rigor in middle school. All students in the district take Common Core Math 6, 7 and 8, a robust foundation that allows them to be more successful in advanced math courses in high school.

8. Banning Times Tables

Professor Boaler's ideas are not without their critics. On a visit to England at the beginning of term, she said that in an ideal world she would ban times tables tests. It's not that times tables aren't worth learning, but the greater goal of maths education is more important, she argues. Therefore, perhaps we should seriously think about whether it is worth risking some children being turned off the subject, aged 8, for the sake of ensuring they can answer 7x6 under pressure, without hesitation.

Even now, the fissure between traditionalists and progressives can erupt. Boaler has plenty of skeptics who think her ideas sacrifice rigor. In 2015, she ignited controversy in Britain by saying at a conference that she had never memorized her times tables. "It has never held me back, even though I work with maths every day," she said. "It is not terrible to remember math facts; what is terrible is sending kids away to memorize them and giving them tests on them, which will set up

this maths anxiety.”

The reaction to her comments was swift. Charlie Stripp, director of the National Centre for Excellence in the Teaching of Mathematics, said: “It is not the learning of times tables that is causing anxiety but rather it is lack of times table knowledge. It should be an educational entitlement that all children are helped to learn their times tables.”

Professor Boaler emphasizes that her views come from decades of academic work. “These are not just ideas,” she says. “We have research data. We’re in a time when other educators outside maths are pushing for more problem-solving, more applied thinking, less testing.

So this article reveals what we found, which is that, when we divide kids by their learning approaches we put them into different classes by the learning approach. Some kids are "memorizers." They think that in math, you just have to memorize lots of methods — every step. And the U.S. has a very large number of "memorizers" — it's in the top third of countries in the world [by that measure].

And what we found is that the kids who memorize are the lowest-achieving students in the world. Inside each country, they are the lowest-achieving students. And every country with high levels of memorizers is a low-achieving country.

So that isn't how you become successful in math. You become successful by seeing that there are just a few big ideas in math that you need to link together and think about in depth. And once you've understood the core ideas in math, everything kind of comes together. There's really very little to remember.

9. Visual Math

When engaged actively to think and talk about ideas, students do better in math.

This sort of work – considering, visualizing and describing patterns is at the heart of mathematics. to find a way of visualizing and representing the pattern, using algebra to describe the changing parts of the pattern – is extremely important algebraic work.

In addition to ideas about neuroplasticity and the openness of mathematics, our mathematical mindset approach includes other research from brain science and education about effective learning.

For example, recent brain science has shown that when people work on mathematics, five different pathways in the brain are involved, and two of them are visual pathways [10,26]. The mathematical mindset approach teaches about the value of different representations in mathematics, with students creating ideas in visual, numerical, verbal, and other forms, encouraging connections between pathways in the brain. Mayer [27] studied the impact of a “multimedia” approach to learning, and found that teaching through different representations, including visuals, deepened understanding for students. Our approach also shares the importance of struggle and of making mistakes for brain growth. At its core, the mathematical mindset approach to teaching focuses on providing all students with access to complex and rich mathematical work that encourages brain connections through multiple representations of ideas.

Some scholars note that it will be those who have developed visual thinking who will be “at the top of the class” in the world’s new high-tech workplace that increasingly draws upon visualization technologies and techniques, in business, technology, art, and science.
The finger research is part of a larger group of studies on cognition and the brain showing the importance of visual engagement with math. Our brains are made up of “distributed networks,” and when we handle knowledge, different areas of the brain communicate with each other. When we work on math, in particular, brain activity is distributed among many different networks, which include areas within the ventral and dorsal pathways, both of which are visual. Neuroimaging has shown that even when people work on a number calculation, such as 12 x 25, with symbolic digits (12 and 25) our mathematical thinking is grounded in visual processing.

In the YouCubed study “Visual Math Improves Math Performance,” Boaler shares:

In a ground breaking new study Joonkoo Park & Elizabeth Brannon (2013), found that the most powerful learning occurs when we use different areas of the brain. When students work with symbols, such as numbers, they are using a different area of the brain than when they work with visual and spatial information, such as an array of dots. The researchers found that mathematics learning and performance was optimized when the two areas of the brain were communicating.

Boaler goes on to say that "Mathematics is a subject that allows for precise thinking, but when that precise thinking is combined with creativity, openness, visualization, and flexibility, the mathematics comes alive."

When your students aren't thinking about math in a visual context, they are missing out on developing a deep understanding of the material.

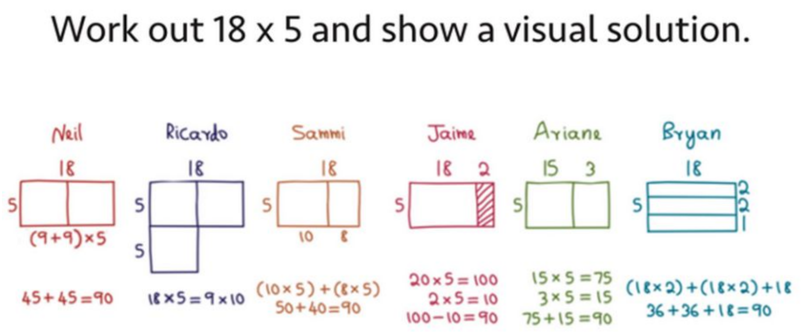
It is hardly surprising that students so often feel that math is inaccessible and uninteresting when they are plunged into a world of abstraction and numbers in classrooms. Students are made to memorize math facts, and plough through worksheets of numbers, with few visual or creative representations of math, often because of policy directives and faulty curriculum guides. The Common Core standards for kindergarten through eighth grade pay more attention to visual work than many previous sets of learning benchmarks, but their high-school content commits teachers to numerical and abstract thinking. And where the Common Core does encourage visual work, it’s usually encouraged as a prelude to the development of abstract ideas rather than a tool for seeing and extending mathematical ideas and strengthening important brain networks.

To engage students in productive visual thinking, they should be asked, at regular intervals, how they see mathematical ideas, and to draw what they see. They can be given activities with visual questions and they can be asked to provide visual solutions to questions. When the youcubed team (a center at Stanford) created a free set of visual and open mathematics lessons for grades three through nine last summer, which invited students to appreciate the beauty in mathematics, they were downloaded 250,000 times by teachers and used in every state across the U.S. Ninety-eight percent of teachers said they would like more of the activities, and 89 percent of students reported that the visual activities enhanced their learning of mathematics. Meanwhile, 94 percent of students said they had learned to “keep going even when work is hard and I make mistakes.” Such activities not only offer deep engagement, new understandings, and visual-brain activity, but they show students that mathematics can be an open and beautiful subject, rather than a fixed, closed,

and impenetrable subject.

By adopting richer, more open teaching methods and encouraging kids to adopt a growth mindset, Boaler believes, educators can help students make strides. Of course, it all unfolded in a university setting, overseen by five teachers, two grad students, 12 undergrads and a staff member — a level of support a typical teacher wouldn't dream of.

By adopting richer, more open teaching methods and encouraging kids to adopt a growth mindset, Boaler believes, educators can help students make strides. In 2015, she and her associates brought 81 middle schoolers — many of them underachievers — to campus for a four-week math camp centered on activities taken from the Week of Inspirational Math. The students began the camp convinced they were "not math people," Boaler says. But they were soon engaged. After four weeks of morning classes and afternoon enrichment, the students had improved their scores on standardized math tests by an average of 50 percent, or 2.7 school years.



from Jo Boaler. Mathematical Mindsets (2016)

The U.S. actually had more memorizers than South Korea, long thought to be the paradigm of rote learning. Why? Because American schools routinely present mathematics procedurally, as sets of steps to memorize and apply. Many teachers, faced with long lists of content to cover to satisfy state and federal requirements, worry that students do not have enough time to explore math topics in depth. Others simply teach as they were taught. And few have the opportunity to stay current with what research shows about how kids learn math best: as an open, conceptual, inquiry-based subject.

In 2005 psychologist Margarete Delazer of Medical University of Innsbruck in Austria and her colleagues took functional MRI scans of students learning math facts in two ways: some were encouraged to memorize and others to work those facts out, considering various strategies. The scans revealed that these two approaches involved completely different brain pathways. The study also found that the subjects who did not memorize learned their math facts more securely and were more adept at applying them. Memorizing some mathematics is useful, but the researchers' conclusions were clear: an automatic command of times tables or other facts should be reached through "understanding of the underlying numerical relations."

Additional evidence tells us that students gain a deeper understanding of math when they approach it visually—for instance, seeing multiplication facts as rectangular arrays or quadratic functions as growing patterns. When we think about or use symbols and numbers, we use different brain pathways than when we visualize or estimate with numbers. In a 2012 imaging study, psychologist Joonkoo Park, now at the University of Massachusetts Amherst, and his colleagues

demonstrated that people who were particularly adept at subtraction—considered conceptually more difficult than addition—tapped more than one brain pathway to solve problems. And a year later Park and psychologist Elizabeth Brannon, both then at Duke University, found that students could boost their math proficiency through training that engaged the approximate number system, a cognitive system that helps us estimate quantities.

“Many parents have asked me: What is the point of my child explaining their work if they can get the answer right? My answer is always the same: Explaining your work is what, in mathematics, we call reasoning, and reasoning is central to the discipline of mathematics.”

10. Finger Math

Teachers should celebrate and encourage finger use among younger learners and enable learners of any age to strengthen this brain capacity through finger counting and use.
Neuroscientists often debate why finger knowledge predicts math achievement, but they clearly agree on one thing: That knowledge is critical. As Brian Butterworth, a leading researcher in this area, has written, if students aren’t learning about numbers through thinking about their fingers, numbers “will never have a normal representation in the brain.”

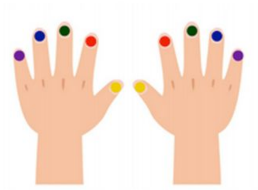
Stopping students from using their fingers when they count could, according to the new brain research, be akin to halting their mathematical development. Fingers are probably one of our most useful visual aids, and the finger area of our brain is used well into adulthood. The need for and importance of finger perception could even be the reason that pianists, and other musicians, often display higher mathematical understanding than people who don’t learn a musical instrument.

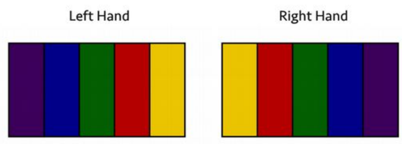
In a study published last year, the researchers Ilaria Berteletti and James R. Booth analyzed a specific region of our brain that is dedicated to the perception and representation of fingers known as the somatosensory finger area. Remarkably, brain researchers know that we “see” a representation of our fingers in our brains, even when we do not use fingers in a calculation. The researchers found that when 8-to-13-year-olds were given complex subtraction problems, the somatosensory finger area lit up, even though the students did not use their fingers. This finger-representation area was, according to their study, also engaged to a greater extent with more complex problems that involved higher numbers and more manipulation. Other researchers have found that the better students’ knowledge of their fingers was in the first grade, the higher they scored on number comparison and estimation in the second grade. Even university students’ finger perception predicted their calculation scores.

We brought 81 kids onto campus last summer and we taught them algebra for 18 days. They were all kids who had had bad experiences. And we taught them algebra through visual creative math. That's how you understand algebra best: you see it. At the end of 18 days, these kids improved their standardized test scores by an average of 50 percent. That's equivalent to 1.6 years of school growth. And they spent 18 lessons with us.

Recent brain science suggests that preventing students from using their fingers when they count could actually hurt their mathematical development.

You can support visual math learning by encouraging finger counting among students and enable them to strengthen their brain capacity in the process.





11. Number Sense

Boaler's argument has several parts. She explains that the key to success in math is having something called "number sense," and number sense is developed through "rich" mathematical problems. Too much emphasis on rote memorization, she says, inhibits students' abilities to think about numbers creatively, to build them up and break them down.

Neither Wolfram nor I are arguing that schools should not teach calculating, but the balance needs to change, and students need to learn calculating through number sense, as well as spend more time on the under-developed but critical parts of mathematics such as problem solving and reasoning.

It is important when teaching students number sense and number facts never to emphasize speed. In fact this is true for all mathematics. There is a common and damaging misconception in mathematics – then idea that strong math students are fast math students

In essence, she found students from the more progressive, "chaotic" school knew less but understood more.

Encourage students to think deeply about mathematics
Students achieve higher when they are exposed to the kind of deep understanding that's typically unavailable in math classrooms.
This is especially true for girls who studies show tend to be put off by procedural math. Boaler suggests getting at deep understanding by using project-based curriculum with real-life applications, giving students lots of opportunities to work together and get hands-on experience.
Boaler explains in her book that if math is taught using deep learning, instead of as performance subject, students can start to see it as important knowledge that empowers them to think quantitatively to solve problems in their work and lives.

The foundation all math students need is number sense—essentially a feel for numbers, with the agility to use them flexibly and creatively (https://www.youcubed.org/what-is-number-sense/).
A child with number sense might tackle 19 × 9 by first working with "friendlier numbers"—say, 20 × 9—and then subtracting 9. Students without number sense could arrive at the answer only by using an algorithm. To build number sense, students need the opportunity to approach numbers in different ways, to see and use numbers visually, and to play around with different strategies for combining them. Unfortunately, most elementary classrooms ask students to memorize times tables and other number facts, often under time pressure, which research shows can seed math

anxiety. It can actually hinder the development of number sense.

12. Homework is Inequitable and Useless

"research has consistently found homework to either negatively affect or not affect achievement"
But maths classrooms have to change in a lot of ways. It's not just about changing messages for kids. We have to fundamentally change what happens in classrooms. And we want kids to have a growth mindset, to believe that they can grow, and learn anything. But it's very difficult to have a growth mindset in maths. If you're constantly given short, closed questions that you get right or wrong, those questions themselves transmit fixed messages about math, that you can do it or you can't. So we have to open up maths questions so that there's space inside them for learning.

So we need to get research out to teachers. We need a revolution in maths teaching. And if you don't believe me, come listen to this kid. He's a middle schooler, and we had worked with his teachers to shift from worksheet math to open math with mindset messages. This is him reflecting on that shift.

A few years ago I taught a summer math camp for sixth and seventh graders using open, creative, visual math tasks. The students were not judged or graded, and the teachers valued and celebrated mistakes and struggle. At the end of the camp, some students described feeling "mathematically free."

Boaler is a subject of the forthcoming documentary The Gatekeeper, directed and produced by Vicki Abeles, who made the influential film Race to Nowhere about the pressure and stress on students caused by testing and overscheduling. Abeles says Boaler surprised her the first time they met by pointing out something that had escaped her attention: Most of the individual students' struggles documented in Race to Nowhere began in math class. In fact, Abeles had been inspired to create the film after the suicide of a girl in her community who, her family suspected, had begun to question her intelligence and self-worth after struggling in algebra.

Eliminate homework! (or at least change the nature of it)
Great news kids! PISA (The Program for International Student Assessment) conducted a survey of 13 million students to study the relationships between homework, achievement and equity, and found that homework is inherently inequitable, and that it "didn't seem to raise achievement for students."
"This is not an isolated finding; academic research has consistently found homework to either negatively affect or not affect achievement," Boaler wrote.

Why? Students from less-privileged families rarely have quiet place to study or do homework at night, and their parents are more likely to be at work and unable to help). They also have fewer study materials such as books and devices with Internet access.
"When we assign homework to students, we provide barriers to the students who need our support," Boaler wrote, "This fact, alone, makes homework indefensible to me."
She suggests that teachers and school leaders who want to promote equity by eradicating homework arm themselves with resources like "The Case Against Homework" by Alfie Kohn and "The One World School House" by Sal Khan, which reinforce these conclusions.
But old habits die hard, and the quantity of homework is often seen as a measure of a curriculum's rigor.

“If you want to retain a homework, then I recommend changing the nature homework,” Boaler suggests. “Instead of giving questions students need to answer in a performance orientation, give reflection questions to encourage students to think back on mathematics lesson and focus on the big ideas.”

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13. Detracking, Group Work and Mixed Ability Teaching

Offer high-level content to all students:
“Put simply, if students spend time in class where they are given access to high-level content, they achieve at higher levels,” Boaler writes. So why not give all students access to advanced math courses?

According to Boaler, radical teachers have achieved similar results by offering advanced math classes to all students, dismissing the myth of “gifted child” which has been used to separate into those with and without math gene into separate courses (an injustice that was actually the subject of a class action lawsuit in California).

“Group work is a strategy I regard as critical to good mathematics work,” Boaler wrote, “But a fascinating study showed that group work may also be critical in countering racial inequities in mathematics achievement and course taking.”
Research shows that black students’ results were better when working together. Back in the ’70s, Uri Treisman of the University of California, Berkeley was alarmed to find 60% of black students failing required calculus courses, despite coming with good GPA’s from high school. Treisman introduced a new approach in which students were offered collaborative workshops so they could work mathematics together. The failure rates dramatically dropped to zero in two years.
More recent research studies by Boaler and others confirm this result.
“When students work on mathematics collaboratively, which also gives them opportunities to see and understand mathematics connections — equitable outcomes results,” she wrote.

Having one core sequence provides focus and coherence as schools and teachers implement the CCSS-M and supports equity by creating one path for all students to experience rigorous

mathematics. We believe that secondary schools do not separate their students into an honors track and a regular track—or into other tracks based on perceived ability—until students choose course pathways at the end of 10th grade.

San Francisco's policy shift began in earnest five years ago. As of last year, low performance among students in middle-school math (getting D's and F's) had been reduced by one-third. The share of students needing to repeat Algebra 1 in high school — the classic pathway to dropping out of math — has declined from 40 percent in 2017 to only 8 percent in 2018. As a result, more students than ever are taking a fourth year of high school math and advanced classes beyond Algebra 2. Going for depth of understanding in the foundational years, and accelerating only when students have solid backgrounds and identified goals, has paid off. This is progress we can't risk undoing by returning to the failed practices of early acceleration.

It's especially promising that gains are being made by the full range of students in the San Francisco schools. Groups that traditionally underachieve — for example, students of color, female students, students of low socioeconomic status, bilingual students and students with special needs — have all experienced increases in achievement. We congratulate the district for its wisdom in building course sequences that serve all students increasingly well.

Currently three fifths of U.S. students fail mathematics, and mathematics is a harshly inequitable subject (Kozol 2012; Silva & White 2013). When our classrooms change—when students are encouraged to believe they can be successful in mathematics and are taught using the high-quality teaching methods they deserve—the landscape of mathematics teaching and learning in the United States will change forever (Boaler & Foster 2014).

Perhaps most significantly and most radically, schools should also remove fixed student groupings that transmit fixed mindset messages and replace them with flexible groupings that recognize that students have different strengths at different times (Boaler 2009; Boaler & Foster 2014).

Separating students by perceived ability level into different classes creates distinct classroom cultures. In lower-tracked classes, students receive more procedural activities and what is reinforced is often behavior instead of academics. In higher-tracked classes, students are often over-accelerated, meaning that they may cover more material but still stay at a surface level. Both conditions are a disservice, in that math is narrowly defined as skills and answer-getting, with students feeling anxiety about performance. Detracking improves the classroom culture for a large number of students who otherwise would not have access to high academic expectations or motivated classmates; this includes students in both the high and low tracks. Additionally, criteria for placing students in high or low tracks have often involved those narrowly defined conceptions of math learning. With the new Common Core Standards for Mathematics, the content standards and the Standards for Mathematical Practices expands what it means to do and learn math.

The SFUSD core curriculum was developed to support heterogeneous classes. Shifting the emphasis away from getting the right answer quickly and toward deeper thinking and discussion opens who feels included and successful, thus decreasing disruptive or apathetic behavior. Tasks and problems are designed to provide access and challenge all students.

When students feel their ideas are valued both by the teacher and fellow students and know they can contribute to whole class learning, there is more buy-in and collaboration. Students come to

see that they can do math and take more responsibility for their own learning and work to support each other by being inclusive and helpful. Students who are a part of a supportive community also take more risks, advocating for their own learning by asking questions to solidify their understanding.

Research has shown that all learners benefit in a heterogeneous setting (including in schools with larger class sizes). The research has been conducted in a variety of settings, including urban districts, in the US, in England, and Australia. All learners benefit from the variety of thinking that is expressed, including more able students who benefit from the need to explain their thinking clearly and concisely. Creating explanations serves to consolidate and enhance their understanding, while other learners benefit from the explanations. All students studied have improved their performance compared to tracked classroom settings. To read more about the research on detracking, see http://www.sfusdmath.org/articles-of-interest.html

The SFUSD core curriculum provides rigorous math tasks that allow access for many types of learners. The tasks and activities are designed to be highly engaging, promote productive struggle, and often have multiple solution strategies. This allows students many opportunities to delve deeper into the mathematics, strengthen their understanding of concepts and explore mathematics related to interesting real-world situations. Rich tasks provide natural opportunities for extensions that students often identify themselves or that the teacher can offer students for deeper investigations that are the heart of a STEM-oriented education.

It's not about going faster, it's about going deeper. The US is suffering from a massive over-acceleration of students in high school that is contributing to a declining rate of students choosing STEM majors once they are in college. The SFUSD core curriculum challenges students who are used to successfully getting the right answer quickly to deepen their understanding by explaining their thinking and understanding other students' thinking. This approach asks students to use multiple strategies and make conceptual connections, which helps develop the types of complex thinking that are called for by the research and business innovation communities.

Students working on challenging tasks in heterogeneous groups develop communication and collaboration skills that go beyond the math content. In heterogeneous groups, students are more likely to experience the benefits of learning together, appreciating different perspectives, and building upon one another's strengths, which makes them better prepared for college and the workplace. Businesses are often looking for employees who are skilled and successful at working in collaboration with others because it allows for innovation and efficiency.

Why is mixed-ability teaching so unpopular in England? I have recently completed a research study in California, in which I studied a mixed-ability maths approach in an urban multicultural school. Students achieved at higher levels than other students, they enjoyed maths more, and the ethnic cliques common in other schools did not form. Class visitors are amazed by the calm environment, with all students on-task, working hard and treating each other respectfully.

The approach was successful for high-attaining students in our four-year study, who achieved more in mixed-ability classes than high-attaining students in other schools who were put into high "tracks". Students told us that they learnt to respect students from different cultures and circumstances. Significantly, the equitable relations students formed came through a maths approach that also brought about extremely high achievement. By their senior year, 41 per cent of

the students, many of whom were from ethnic minorities and low-income homes, were in calculus classes (similar to A-level maths in England), such was the love of maths they developed.

Longer accounts of the approach that the school used are available elsewhere, but one feature I want to highlight is the multi-dimensionality of the classrooms. Multi-dimensional classrooms are those in which teachers value many dimensions of mathematical work, not just the execution of procedures. Students work on open-ended problems that provide different access points and solution paths, and that allow for multiple representations. As students work on the problems, they are rewarded for such activities as asking good questions, rephrasing problems, explaining ideas, being logical, justifying methods, or bringing a different perspective to a problem.

Put simply, there are many more ways to be successful, so that many more students are successful. This mixed ability, multi-dimensional approach means that success was an option for all students. It gives them access to university and to higher-level jobs, and it allows many to plan mathematical careers. Such results are not usually achieved in urban schools in the US or England. I hope that English schools may benefit, too, from this approach, though I fear that UK politicians' blind commitment to setting will make this difficult.

14. Why Common Core is Good

Too often, it has been "plug and chug" math. In this approach, math is a bunch of memorized rules that don't make much sense. Follow the rules, and you will get the right answer. Do something different, and you're likely to get it wrong. "Analytical thinking" consists of figuring out which rule to apply. There is limited need for originality, explanations, or even genuine understanding. Learning enough rules will allow you to solve the problems you are given. Do this for enough years, and you may firmly believe that this is what mathematics actually is. If your kids are asked to do something different, you may be up in arms.

Math as rules starts early. Kids learn in elementary school that you can "add a zero to multiply by ten." And it's true, 237 x 10 = 2370. Never mind why. But then when kids learn decimals, the rule fails: 2.37 x 10 is not 2.370. One approach is to simply add another rule. But that's not the best way.

Common Core saves us from plug-and-chug. In fact, math is based on a collection of ideas that do make sense. The rules come from the ideas. Common Core asks students to learn math this way, with both computational fluency and understanding of the ideas.

Learning math this way leads to deeper understanding, obviates the need for endless rule-memorizing and provides the intellectual flexibility to apply math in new situations, ones for which the rules need to be adapted. (It's also a lot more fun.) Combining computational fluency with understanding makes for problem solvers who can genuinely use their math. This is what businesses want and what is necessary to use math in a quantitative discipline.

Here is what good math learning produces: Students who can compute correctly and wisely, choosing the best way to do a given computation; students who can explain what they are doing when they solve a problem or use math to analyze a situation; and students who have the flexibility and understanding to find the best approach to a new problem.

Common Core promotes this. It systematically and coherently specifies the topics and connections

needed for math to make sense, and promotes both understanding and accuracy.

There is no doubt that the new standards are more rigorous. They will require more of our students, our teachers and our parents. Knowing what you are doing, instead of just knowing a set of rules, is the essential foundation for applying math to the real world.

The Common Core provides a set of standards that are focused, coherent, and rigorous. Focusing deeply on fewer concepts allows students to gain strong foundational conceptual understanding. Developing coherence across grades allows students to build upon deep conceptual understanding from earlier years so that each standard is not a new event, but an extension of previous learning. All students should take rigorous courses that balance conceptual understanding—the ability to access concepts from multiple perspectives, discuss them, and apply them to new situations—with procedural skill and fluency.

The Common Core does not eliminate computation, calculation, practice, or homework. Rather it seeks a balance in which conceptual understanding is not sacrificed for memorizing procedures. There is an emphasis on enabling students to make sense of the math and seeing math as a way to solve real life problems. Students become better problem solvers when they understand both the problem and the concepts involved in solving it. In the traditional approach, many students have memorized formulas but don't know when to apply them. Students who develop a deep understanding of mathematics can derive the formula when they need it, know when they need it, and don't apply the wrong formula to a situation because it's the one they have memorized. Homework and practice are still valuable and teachers should have a system to capture their students' progress, but teachers might rethink the purpose of homework, perhaps scaling down the number of problems with the expectation that students show their thinking and demonstrate understanding using multiple representations.
The process of creating a core curriculum began two years ago before there were textbooks truly aligned to the Common Core standards. The elementary core curriculum still makes significant use of the adopted Elementary Math textbooks, and the secondary core curriculum makes significant use of the College Preparatory Mathematics (CPM) textbooks, which are now aligned to the Common Core. SFUSD has made an agreement with CPM to provide all SFUSD students with free eBook access.

The Common Core emphasizes conceptual understanding along with procedural fluency. While the goal of learning procedures used to be application (e.g., learning to multiply in order to do bookkeeping), we now have calculators and computers for that purpose. Now the goal of learning algorithms and math strategies is to illuminate the number system and make connections between mathematical concepts. It is also very important to understand procedures conceptually and flexibly (for example, When do I divide? How can I figure this out with a visual diagram or with an equation?) in order to solve problems in the real world.

Why Students in the US need Common Core Math: https://www.youtube.com/watch?v=pOOW0hQgVPQ , and see http://nrich.maths.org.)

15. **Postponing Algebra to High School**

The Common Core describes a progression of algebra from Kindergarten through Grade 8 that leads to the CCSS Algebra course in high school. CCSS Math 8 introduces extensive new mathematics

content traditionally taught in high school—linear functions, transformational geometry, and bivariate statistics. CCSS Algebra and CCSS Geometry are built upon the extensive development of the core concepts in CCSS Math 8 (linear functions and equations, transformational geometry), so 8th grade students will be prepared for high school courses.

The SFUSD course sequence is based on a belief that students learn best in heterogeneous classes that hold high expectations for all students. The curriculum was created to address standards that spiral or progress through the grade levels. The 8th grade curriculum focuses on formulating and reasoning about expressions and equations, grasping the concept of functions and using functions to describe quantitative relationships, and analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence. These are critical foundational skills in preparing students for algebra, geometry, and statistics in high school.

Algebra 1 was traditionally a high school course that only a small number of students took in middle school, but over the last 15 years there has been a push to accelerate increasing numbers of students (or in the case of California, all students) into middle school Algebra 1. As a result, record numbers of students are failing and repeating Algebra 1, especially students from underserved communities. By moving Algebra 1 back into 9th grade for all students and replacing it with CCSS Math 8—a course that explicitly develops concepts needed for success in Algebra—students will experience more confidence and success because they have time to do mathematics with each other, discussing their learning, examining each other's work, and building a deeper understanding of concepts.

After 10th grade, students can choose to take an Honors Algebra 2 course that compresses CCSS Algebra 2 with Precalculus, which allows them to take AP Calculus in 12th grade. Unlike the earlier practice of having students accelerate in math by skipping a course, the Common Core necessitates that acceleration only occur by course compression—learning the standards from more than one year during a regular class period over one year. The option for compression supports students who wish to graduate from high school prepared for specialized studies in STEM in university settings.

The middle-school courses provide the indispensable building blocks upon which a solid foundation is established. If any of these blocks is missing, the foundation is weakened. Skipping or accelerating through these courses would be harmful. The current program provides much more success for students interested in STEM fields.

There is a common misunderstanding that the Algebra 1 course taught under No Child Left Behind was the same course that is currently taught in the city schools. The Common Core State Standards raised the level and rigor of eighth-grade mathematics to include Algebra 1 content as well as geometry and statistical topics previously taught in high school. It is fair to say the content of the district's eighth-grade math course was college-prep, high-school-level math for most of the current students' parents. The current Algebra 1 course is more conceptually demanding and requires that students have the foundational background of the math taught in eighth grade.

16. Displacing Algebra with Data Science

Thanks to the information revolution, a stunning 90% of the data created by humanity has been generated in just the past two years.

Yet the math taught in U.S. schools hasn't materially changed since Sputnik was sent into orbit in the late 1950s. Our high school students are taught algebra, geometry, a second year of algebra, and calculus (for the most advanced students) because Eisenhower-era policymakers believed this curriculum would produce the best rocket scientists to work on projects during the Cold War.

It has been 50 years since the U.S. reached the moon, almost 30 years since the Berlin Wall fell. Technology has advanced to the point that tiny powerful computers are routinely carried around in pockets and purses. Times have changed, and so has the math people use in everyday life.

We surveyed 900 "Freakonomics" podcast listeners — a pretty nerdy group, we must admit — and discovered that less than 12% used any algebra, trigonometry or calculus in their daily lives. Only 2% use integrals or derivatives, the foundational building blocks of calculus. In contrast, a whopping 66% work with basic analytical software like Microsoft Excel on a daily basis.

When was the last time you divided a polynomial? If you were asked to do so today, would you remember how? For the most part, students are no longer taught to write cursive, how to use a slide rule, or any number of things that were once useful in everyday life. Let's put working out polynomial division using pencil and paper on the same ash heap as sock darning and shorthand.

What we propose is as obvious as it is radical: to put data and its analysis at the center of high school mathematics. Every high school student should graduate with an understanding of data, spreadsheets, and the difference between correlation and causality. Moreover, teaching students to make data-based arguments will endow them with many of the same critical-thinking skills they are learning today through algebraic proofs, but also give them more practical skills for navigating our newly data-rich world.

Data-based math courses allow students to grapple with real-life problems. They might analyze issues about the environment, space travel or nutrition. Students can examine the threat of wildfires or the ways social media is tracking their data, learning how to apply math to real-world issues.

Other countries are moving much faster than the U.S. in instituting such a curriculum. Over the last 50 years, statistics and data science have become an integral part of the United Kingdom curriculum. Canada's educational system, which is ranked highly internationally, also incorporates statistics and data.

In addition, the Program for International Student Assessment, or PISA, measures how effectively countries are preparing students for the mathematical demands of the 21st century. Last week, PISA released a mathematics framework that guides the assessments. Data literacy is central to the framework. In contrast, U.S. high school students learn algebra and geometry — and are woefully underprepared for the modern world.

The Los Angeles Unified School District is leading the way in updating the way math is taught. In 2013, the LAUSD secured approval from the University of California to recognize data science as a statistics course that students can substitute for Algebra 2 in the college pathway. Over 2,000 students are taking advantage of this option.

For this revolution to be carried out across the country, decision makers will need to hear from

parents and other interested parties who recognize that our children deserve math instruction that is relevant to their lives.

Section IV. Understanding Jo Boaler's Perspectives Through Historical Lens

Progressive education has burgeoned and spread to US schools since the early 20th century. Its origin can be traced back to Jean Jacques Rousseau, the Enlightenment thinker who championed collectivism (subordinating an individual's freedom to national interests), radical equality (equal socioeconomic outcome, rather than equal opportunity, for all), and direct democracy (in contrast to representative democracy). Rousseau's collectivism inspired John Dewey's "school-to-work" philosophy that calls for aiming public education at producing the future workforce. Rousseau's utopian idealism of radical equality summoned generations of progressive educators to pursue equal academic outcomes for all students. Rousseau's naturalistic, child-centered education was the prototype and essence of all sorts of progressive education innovations by his disciples in the past and today.

America's pioneer progressive educators, such as Lewis Terman, Ellwood Cubberley, and Edward Thorndike, misbelieved that females and minorities have a lower IQ and think differently than white males; thus, they could not master advanced math. Such racial and gender prejudices compel progressive educators to equate traditional math with elitism and to lower academic standards for achieving academic equity and social justice.

Based on his animal experiments, Thorndike claimed that training in a specific task would not boost one's overall learning ability, suggesting the study of math would not develop students' mental faculties. Drawing upon Thorndike's various theories, William Kilpatrick and others preached that any math education should be for purely utilitarian purposes while strictly limiting those "useless" academic contents.

Thorndike's connectionism theory stated that human learning behavior is a product of the connection of numerous neural and psychological processes. Progressive educators applied this theory to justify slow-paced, fragmented, and spiraling math curricula, for fear that teaching closely related ideas too close in time might establish incorrect connection or "bonds."

Hence, during the early 20th century, out of the confluence of collectivism, egalitarianism, utilitarianism, pragmatism, social efficiency pursuits, and racial and gender prejudices had outgrown the American progressive education paradigm that features low expectations and lax standards for student academic achievement. From then on, American schools have become "educational expert's paradises" to experiment with one educational fad after another, such as "learning-by-doing," "critical thinking" and "project method," "higher-order thinking," "discovery learning," "constructivism," "project-based learning," "personalized learning," "deeper learning," "brain science," "21st century skills," and so forth. Essentially the descendants and variants of Rousseau's naturalistic, child-centered education doctrines, these immensely appealing progressive innovations have been employed to dismiss learning basic facts and skills as "rote memorization" or "lower-order thinking" and to advance progressive pedagogy as "higher-order thinking" that offers "conceptual understanding." Over the course, US K-12 math has been transformed into a slow-paced, fragmented, incoherent, and spiraling structure featuring deficient math content and exuberant progressive elements.

The nutshell review above enables us to navigate through the maze of the "same old, same old" perspectives offered by old and new progressive educators.

1. **Traditional Math is Impractical and Useless**

William Kilpatrick:

> The study of algebra and geometry in high school should be discontinued except as an intellectual luxury. Mathematics is harmful rather than helpful to the kind of thinking necessary for ordinary living.

David Snedden:

> Algebra...is a nonfunctional and nearly valueless subject for 90 percent of all boys and 99 percent of all girls--and no changes in method or content will change that.

NCTM:

> The NCTM 1980 publication, *An Agenda for Action*, called for "Mathematics educators and college mathematicians" to "reevaluate the role of calculus in the differentiated mathematics programs." The de-emphasis of calculus would support the move away from the systematic development of the prerequisites of calculus: algebra, geometry, and trigonometry. The so-called "integrated" high school math books containing parts of algebra, geometry, and trigonometry are not systematic, and often depended on student "discoveries" that were incidental to solving "real world problems."

Marc Tucker:

> Fewer than five percent of American workers and an even smaller percentage of community college students will ever need to master the courses in this sequence in their college or the workplace. For most of our students, those 'high' standards in mathematics constitute a requirement to learn material they will never need, either in college or later in their work, a bit like the requirement a century ago to learn Latin in high school.
>
> The brain's capacity is enormous, but it is not unlimited. Why use up that capacity for the storage of knowledge you will not use? Better to have a use-it-or-lose-it policy that reserves our brain capacity for things that we really need. … Our brains contain the capacity to make literally billions of connections and it is those connections that enable us to think and do. Synapses connect neurons together to make these connections. Patterns of connections that are made repeatedly and often are strengthened; those that are not used wither.
>
> It makes no sense to rush through the middle school mathematics curriculum in order to get to advanced algebra as rapidly as possible. Given the strong evidence that mastery of middle school mathematics plays a very important role in college and career success, strong consideration should be given to spending more time, not less, on the mastery of middle school mathematics, and requiring students to master Algebra 1 no later than the end of their sophomore year in high school, rather than by the end of middle school.

Jo Boaler, Steve Levitt and David Coleman:

> We surveyed 900 "Freakonomics" podcast listeners — a pretty nerdy group, we must admit — and discovered that less than 12% used any algebra, trigonometry or calculus in

their daily lives. Only 2% use integrals or derivatives, the foundational building blocks of calculus. In contrast, a whopping 66% work with basic analytical software like Microsoft Excel on a daily basis.

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2. **Traditional Math is Elitism, Racism and Sexism and Barrier to Social Justice**

Alan Schoenfeld:

... lack of access to mathematics is a barrier – a barrier that leaves people socially and economically disenfranchised. The traditional curriculum bore the recognizable traces of its elitist ancestry: The high school curriculum was designed for those who intended to pursue higher education.

If all students do not have an opportunity to learn this mathematics, we face the danger of creating an intellectual elite and a polarized society. The image of a society in which a few have the mathematical knowledge needed for the control of economic and scientific developments is not consistent either with the values of a just democratic system or with its economic needs.

The Standards, buttressed by NCTM's call for 'mathematics for all' and the equity agenda in Everybody Counts, clearly sat in the education-for-democratic-equality [camp]. . In contrast, . . . the traditional curriculum was a vehicle for . . . the perpetuation of privilege. . .Thus the Standards could be seen as a threat to the current social order. . . the traditional curriculum, with its filtering mechanisms and high dropout and failure rates (especially for certain

minority groups) has had the effect of putting and keeping certain groups "in their place."

Genethia Hayes:

I will advocate as hard as I can with my colleagues to make sure this particular door never gets shut for children of color. I really do see this as an issue of social justice.

Jo Boaler:

Some people revel in the inaccessibility of mathematics as it's currently taught, especially if their own children are succeeding, because they want to keep clear a societal advantage… Mathematics has the greatest and most indefensible differences in achievement and participation for students of different ethnicities, genders, and socioeconomic income levels for any subject taught in the United States.

My main point is that it may not be enough, as a math teacher, to treat students equally in the pursuit of equity. Some students face additional barriers and disadvantages, and we must work to address those quite deliberately if we are to achieve a more equitable society.

There are also political undercurrents to keeping the system as it is now - some people don't want teaching to be more equitable. They do not want to open the doors to everyone."

We know that the way math is taught now appeals to boys more than girls. Particularly the traditional teaching and rules - boys can turn that into competition. Girls, more than boys, seek to understand deeply, to make connections between math and the real world. They seek a more connected approach to the subject. And teaching in more connected ways enables both boys and girls to do better. In the absence of that, it is instead more procedural.

But math classrooms have to change in a lot of ways. It's not just about changing messages for kids. We have to fundamentally change what happens in classrooms. But it's very difficult to have a growth mindset in maths. If you're constantly given short, closed questions that you get right or wrong, those questions themselves transmit fixed messages about math, that you can do it or you can't. So we have to open up maths questions so that there's space inside them for learning.

When we assign homework to students, we provide barriers to the students who need our support. This fact, alone, makes homework indefensible to me.

But a fascinating study showed that group work may also be critical in countering racial inequities in mathematics achievement and course taking.

Perhaps most significantly and most radically, schools should also remove fixed student groupings that transmit fixed mindset messages and replace them with flexible groupings that recognize that students have different strengths at different times.

3. Learning Standard Algorithms and Basic Skills is Harmful

Constance Kamii:

By the 1980s, some researchers were seriously questioning the wisdom of teaching conventional algorithms. Some investigators went further in the 1990s and concluded that algorithms are harmful to children. Piaget's constructivism, and the more than sixty years of scientific research by him and others all over the world led her to a compelling hypothesis: Children in the primary grades should be able to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks. This hypothesis was amply verified.

Ruth Parker:

Long division and multiplication tables are nonsensical leftovers from a pre-calculator age. Let kids play with numbers, and they will figure out most any math concept.

Marilyn Burns:

I am a teacher who has embraced the call for change completely. I've made shifts in my teaching so that helping children learn to think, reason, and solve problems has become the primary objective of my math instruction...I do not give timed tests on basic facts. I make calculators available for students to use at all times. I incorporate a variety of manipulative materials into my instruction. I do not rely on textbooks because textbooks, for the most part, encourage "doing the page" rather than "doing mathematics."

National Science Foundation (NSF):

All children can learn by using and manipulating scientific and mathematical ideas that are meaningful and relate to real-world situations and to real problems. Mathematics and science are learned by doing rather than by passive methods of learning such as watching a teacher work at the chalkboard. Inquiry-based learning and hands-on learning more effectively engage students than lectures. The use and manipulation of scientific and mathematical ideas benefits from a variety of contributing perspectives and is, therefore, enhanced by cooperative problem solving. Technology can make learning easier, more comprehensive, and more lasting. Children should learn through group-based discovery with the help of manipulative s and calculators.

Jo Boaler:

The version of math taught in many school classrooms is “narrow and impoverished”, focused on coming up with the right answers quickly. They were not making sense of the math; they were just following procedures to get the right answers. They were basically computers when you punch in a number sentence and an answer comes out.

4. Basic Skills are Outdated by the Need for Problem Solving and Other More Useful Skills

NCTM & NSF:

All students should have access to calculators and increasingly to computers throughout their school mathematics program; It is dangerous to assume that skills from one era will suffice for another; The use of calculators has radically reduced the demand for some

paper-and-pencil techniques. Teachers should decrease emphasis on such activities as...performing paper and pencil calculations with numbers of more than two digits. The new technology not only has made calculations and graphing easier, it has changed the very nature of mathematics...appropriate calculators should be available to all students at all times.

Requiring complete mastery of skills before allowing participation in challenging problem solving is counterproductive; Difficulty with paper-and-pencil computation should not interfere with the learning of problem-solving strategies; Technology would make problem solving available to students without basic skills.

A SCANS Report for America 2000 concludes that students must develop a new set of competencies and new foundation skills. It stresses that skills must be learned in context, that there is no need to learn basic skills before problem solving, and that we must reorient learning away from mere mastery of information toward encouraging students to solve problems.

Traditional notions of basic mathematical competence have been outstripped by ever-higher expectations of the skills and knowledge of workers . . . employees must be prepared to understand the complexities and technologies of communication, to ask questions, to assimilate unfamiliar information, and to work cooperatively in teams. Businesses no longer seek workers with strong backs, clever hands, and "shopkeeper" arithmetic skills.

The 1985 California Model Curriculum Standards, Grades 9-12:

The mathematics program must present to students problems that utilize acquired skills and require the use of problem-solving strategies. Examples of strategies that students should employ are: estimate, look for a pattern, write an equation, guess and test, work backward, draw a picture or diagram, make a list or table, use models, act out the problem, and solve a related but simpler problem. The use of calculators and computers should also be encouraged as an essential part of the problem-solving process. Students should be encouraged to devise their own plans and explore alternate approaches to problems.

5. Reform Math Surpasses Traditional Math in Developing Students' Conceptual Understanding, Higher-order Thinking and Growth Mindsets

Marc Tucker:

A recent addition to the dominance of attention to procedures in K-12 mathematics education accompanied by lack of focus on conceptual understanding contributes significantly to students struggling with middle school mathematics and early algebra in college.

Whatever students did to pass mathematics courses in middle school, it does not appear to require learning the concepts in any durable way. While they may have been taught the appropriate procedures for solving certain standard problems, the high rates of non-completion by the significant percentages of students who arrive at college with the most modest command of mathematics suggest that there are significant weaknesses in

teaching the concepts on which these procedures are based.

Judging by the tests community college teachers administer to their students in the introductory program courses in their career majors, their courses are typically pitched to the lower set of expectations described by Bloom's hierarchy—memorization of facts and mastery of procedures—and not to the kinds of analytical skills, writing ability, ability to synthesize material to put together solutions to problems the student has not seen before, and other complex skills that employers are now demanding.

Jo Boaler:

As a researcher, teacher and evangelist, Boaler is a leading voice for a wholly different pedagogy where speed is out, depth is in, and the journey to an answer can be as important as the destination. It's an approach where sense-making matters more than memorization and retaining "math facts" matters less than understanding how such facts interconnect.

Research has recently shown something stunning—when students make a mistake in math, their brain grows, synapses fire, and connections are made; when they do the work correctly, there is no brain growth. This finding suggests that we want student to make mistakes in math class and that students should not view mistakes as learning failures but as learning achievements.

Students do not, as many assume, need to revisit a mistake and correct it to experience brain growth, although that is always helpful; brain growth comes from the experience of struggle. When students struggle with mathematics, their brains grow; being outside their comfort zone is an extremely important place to be.

When students engage in interesting, challenging mathematics, they see the difference between deeply exploring math and following procedures outlined in traditional textbooks. With a curriculum that inspires a growth mindset and teacher training to support this understanding, students will have opportunities to access deep mathematics learning.

When your students aren't thinking about math in a visual context, they are missing out on developing a deep understanding of the material. It is hardly surprising that students so often feel that math is inaccessible and uninteresting when they are plunged into a world of abstraction and numbers in classrooms. Students are made to memorize math facts, and plough through worksheets of numbers, with few visual or creative representations of math, often because of policy directives and faulty curriculum guides.

To engage students in productive visual thinking, they should be asked, at regular intervals, how they see mathematical ideas, and to draw what they see. They can be given activities with visual questions and they can be asked to provide visual solutions to questions. Such activities not only offer deep engagement, new understandings, and visual-brain activity, but they show students that mathematics can be an open and beautiful subject, rather than a fixed, closed, and impenetrable subject.

In essence, Boaler found students from the more progressive, "chaotic" school knew less but understood more.

6. Traditional Math and Timed Tests Cause Math Anxiety and Fixed Mindsets

NCTM:

An Agenda for Action also called for "a wider range of measures than conventional testing."

Jo Boaler:

When students spend all their time in math class answering discrete questions to which the answers are either right or wrong, it is very difficult to develop a growth mindset or to believe that mathematics is about growth and learning. Math was never meant to be taught by boring, procedural, and rote-learning calculations that are then timed and corrected.

And then we also know the speed and time pressure is damaging for the brain. So when we give kids timed tests, it shuts down the part of the brain that we need to use when they are taking timed tests.

When questions are narrow and closed, i.e., with one correct answer, students are likely to become frustrated if they do not know how to succeed. Over time, they come to see mathematics as a fixed and closed subject. When questions are opened to become low floor and high ceiling, with many entry points and where many different ways of working are valued, students are more likely to experience success and to see mathematics as an open and growth subject.

Teachers always know how well kids are doing, so you really don't need to test them. You really easily have teachers write down what kids know and can do. The kids themselves can also self-assess and tell if things are strong or not. They do that with extreme reliability. You can ask kids make a project, if you want, that tells us about what they know and can do.

And most tests used do not assess what's important anymore. They might assess whether you are computationally fast — but that's the one thing computers do and we don't need humans for.

Starry, Starry Night

-- to mathematicians who have fought for their whole life to salvage U.S. K-12 math

Starry, starry night
Paint your palette blue and gray
Look out on a winter's day
With eyes that know the darkness in my soul

You lamented about the absurdity
Leading astray U.S. K-12 math
You anguished over the fads
That caused the math-science death march

Now I understand
What you tried to say to me
And how you suffered for your sanity
And how you tried to set them free

They would not listen, they're not listening still
Perhaps they never will …

The Road Taken by Johnny Who Can't Calculate

Two roads diverged in a yellow wood,
And sorry Johnny could not travel both.
And be one curious kid, long Johnny stood,
And looked down both as far as he could.

One guided by mathematicians, who urge
Rigor, focus, and coherence.
Additions, subtractions, multiplication tables, and long divisions;
Ratios, rates, percentages, and proportions.
Paper-and-pencil algorithms,
Steadily sharpen your thoughts.
Practices dispel anxiety, and practices grow knacks;
Fears will disappear; confidence will grow.
Knowledge is power, and you earn it with sweat.

The other favored by educational experts, who chant
A child-friendly wonderland:
Story-telling, finger plays, and diagram visuals,
Geometric slides, turns, and flips.
Let calculators do the chores,
And sweetie you are for creativity.
Practices cause anxiety, and practices make you a nerd.
Multiplication tables numb your brains,
Multiple ways for five times ten are the magic.
Spiraling through the K-12 woods, and you gain
Critical thinking, problem-solving, and higher-order thinking.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and Johnny—
Johnny took the one guided by educational experts,
And that has made all the difference.

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