Abstract

This essay begins with a brief summary of the IEA comparisons of mathematics achievement from 1964 to 2007, showing the predominance of Chinese Taipei, Hong Kong, Japan, Korea, and Singapore. Then in response to one scholar’s suggestion that their success stems from the fact that their curricula are challenging, demanding and rigorous, I provide evidence that in the case of Japan, the exact opposite is true, and that the real key is that Japanese elementary teachers use extremely effective teaching methods which they have developed over decades of classroom research. They also understand that their students, who are taught in mixed ability classrooms, need to move at a moderate pace and have frequent periods of vigorous activity. This results in a high level of learning compared to that of students in other systems, which leads to the mistaken assumption that their curriculum is more difficult. An appendix is provided with excerpts of lessons from two different publishers, separated by seven years, illustrating the efficiency and consistency of their teaching practices.
The Illusion of Rigor

by Daniel M. Stamm

International comparisons of mathematics achievement were begun in 1964 by the International Association for the Evaluation of Educational Achievement (IEA). Over a period of almost four decades, the highest achieving systems have been Japan, and, as they successively joined the study, Korea, Singapore Hong Kong and Taiwan. In 1964, Japan was virtually tied with Israel for first place and retained that rank on the Second International Mathematics Study (SIMS) in 1981, while Israel had dropped to 12th place. In 1995, on the Third International Math and Science Study (TIMSS), Japan had dropped to 3rd place and then to 5th place at the 8th grade level in 1999, 2003 and 2007. At the 4th grade level their scores were 3rd and 4th in 2003 and 2007, respectively (4th grade was not tested in 1999). (See Appendix A for links to the history of the IEA and score data.)

What can account for the superior performance of Japan and the other Asian education systems? In three publications, William Schmidt has suggested that it is because their curricula, compared to those in the U.S., are focused, coherent and rigorous, (Schmidt et al 2002; Aharoni 2005, p. 11; Schmidt 2008). Focus refers to covering a small enough number of topics to learn them thoroughly in the time available; coherence denotes teaching the topics in a logical sequence (i.e., not covering advanced topics before more elementary ones have been mastered); and rigor simply means that the material covered is challenging or demanding.

The purpose of this essay is to show that in the case of Japan, rigor has nothing to do with the success of their students in mathematics. The real key is that Japanese elementary teachers have developed extremely effective teaching methods and constantly take into account the basic physical and intellectual needs of their students. In addition, they have great influence in key areas of the Japanese education system.

The Central Role of Classroom Teachers

High-quality teaching is practiced throughout Japan because experienced elementary teachers play a crucial role in the following parts of the Japanese system:

1) the training of new teachers,
2) the professional development of established teachers, through teacher-initiated research groups,
3) classroom research leading to the discovery and implementation of much of the knowledge that those teachers acquire, and
4) the writing of the elementary textbooks and teachers’ manuals that embody their pedagogical technology.

New teacher training—

“The training of Japanese teachers is not thought to begin until they start their first teaching job, at which point they begin a long period of apprenticeship-like training in which they are supervised closely by master teachers.” (Stigler et al, 1996, p. 217). Until 1989, the practice was informal but widespread, and in that year it was formally structured by the Ministry of Education:
The in-service training system for beginning teachers is composed of two parts: apprenticeship training in a school (about seventy days a year) and lecture courses in teacher training centers (about thirty-five days a year). Both use traditional training styles. In apprenticeship training, the first-year teachers are mentored by a retired teacher or a veteran in the school in order to master teaching skills. (Sato 1992, pp. 158-159).

In-service study activities—
Since the early 1950s, research, or study groups allowed for the dissemination of the knowledge base throughout the teaching corps:

[T]he national survey of the National Institute for Educational Research in 1951 indicated that over 80 percent of teachers participated in developing their own school curricula... The movement led to professional autonomy, and teachers organized innumerable voluntary study groups both inside and outside of the schools. The Japan Teachers’ Union, established in 1947, also enhanced teachers’ autonomy by promoting voluntary studies. The union held annual study meetings for teachers at national, prefectural, and local levels, in which teachers developed their professional culture by sharing their practical experience and principles with each other. (ibid., p. 161)

…Both grassroots, teacher-initiated study circles and publicly supported study groups dot Japan's educational landscape. … 'Research' in this context means classroom-based efforts to improve instructional approaches,... attempting to reshape classroom instruction in keeping with [an] idea, and sharing the resulting practices with colleagues (by visiting each other's classrooms, videotapes or reports). (Lewis & Tsuchida 1997, p. 319)

Lesson Study—
The classroom-based research referred to above is known as “lesson study.” According to Nobuo Shimahara, it is “… a widespread popular practice embedded in the culture of teaching, an ethos that Japanese teachers cherish as a proven means to improve teaching.” (Shimahara 2002a, p. 114). The procedure involves an experimental lesson which is developed by a group of teachers over the course of the school year. It is then taught by one of the group members and is carefully observed by the other members, who make notes about how students respond to it and who also make audio and/or video recordings. After it is taught to the class, the teachers spend several hours discussing the results in detail. (Lewis & Tsuchida 1998) Depending on its success, the lesson may be redesigned and retaught, but whatever the eventual outcome, it is written up and published, either for local or wider consumption. Lesson study is such a universal practice that “…journal articles by teachers about their educational research outnumber by a third those of university educational researchers in Japan.” (Sato & McLaughlin 1992, p. 362).

Since lesson study involves designing, testing and evaluating methods of teaching content to students in the classroom, it has produced extremely effective techniques for teaching each topic. It has been applied to the order in which topics are taught and the ages at which students can best learn them. Because Japanese teachers have determined the time necessary for the activities used to teach each topic, they know the maximum number that can be addressed in a given time period. This, plus the correct sequence of topics, give their curriculum its characteristic focus and coherence.
Teacher authorship of elementary texts and teachers’ manuals—

Because experienced teachers have developed and maintained the Japanese teaching knowledge base, they are hired by publishing companies as the principle authors of elementary textbooks. This is reflected in the statements of the informants in a study by Catherine Lewis et al (2002):

Interviewer: Are the people who actually write it more often classroom teachers or university professors?

Mr. Hajime: Classroom teachers. Unless you are a classroom teacher, you can't write it, can you? … University professors might be good at writing, and might be able to write textbooks for junior high or high school, but not textbooks for elementary school. That’s why it is important that elementary teachers participate in the process.

Yet another Japanese author commented: "It's actual teachers who are central, and university professors who are secondary in the textbook writing." (p. 57)

… in the words of one interviewee, "There is a recognition on the part of the national government that teachers are essential to the creation of textbooks. You can't make textbooks without teachers." (p. 58)

Similar teacher involvement prevails in regard to Japanese teachers’ manuals:

…The manuals, usually written by expert teachers in a user-friendly format, provide a coherent body of subject-matter knowledge and specific pedagogical suggestions for teaching the textbook content. These suggestions often involve activities from research lessons or regular classes that demonstrate their effectiveness in guiding students to meet the particular lesson goals. (Lee & Zusho 2002, pp. 68-69)

Reservoir of Teaching Technology

The four features of the Japanese system delineated above demonstrate that the knowledge base of teaching technology, i.e., of the math content appropriate for children at a given age, its correct sequencing and the best methods for teaching it, resides among classroom teachers, not university researchers. The training of Japanese elementary teachers is not characterized by advanced coursework in mathematics or teaching methods. In 1997 less than 5% of elementary teachers had master’s degrees. (Shimahara 2002b, p. 57). The primary source of their knowledge of mathematics and methods for teaching is not college coursework, but their teachers’ manuals (Lee & Zusho 2002, pp. 79-81) and interaction with veteran teachers.

The Illusion of Rigor

Rigor was identified as a characteristic of the curricula of high-achieving systems because “…[I]n the middle grades, the rest of the world is teaching algebra and geometry. The U.S. is still, for most children, teaching arithmetic...[O]ther countries outperform us in the middle and upper grades [1995] because their curricular expectations are so much more demanding, so much more rigorous.” (Schmidt 2008, p. 23). The reality is, however, that rigor plays no part. For the Japanese the preparation for geometry in the eighth grade began in the first grade and continued at levels appropriate for the students’ developmental stages throughout elementary school. Not surprisingly, they outperformed the U.S. in the fourth grade, too, ranking 3rd against
our 12th place. In the 8th grade the rankings were 3rd and 28th, respectively. There is clearly a difference between our systems, but it has nothing to do with rigor.

There are features of Asian instruction which generally enhance the effectiveness of their education by their distinct lack of rigor. These are 1) a marked lack of pressure on students during their classes, and 2) frequent, substantial periods of vigorous physical activity between classes. Regarding the first, James Stigler noted that “…Japanese classrooms appeared to move at a more relaxed pace than American classrooms. Only teachers in Japan were ever observed to spend an entire forty-minute lesson on one or two problems… Understanding takes time, and perhaps spending that time at an early stage will lead to future benefits..” (Stigler, 1988, p.29). The frequency of recesses was noted by Stevenson and Lee: “The school day was highly structured in Taipei [Taiwan] and Sendai [Japan], with 40-45 minutes of class followed by a 10-15 minute break…. [There were an] average of four recesses at the first grade in Sendai and Taipei and of eight at the fifth grade in Taipei and five in Sendai.” (Stevenson and Lee 1990, p. 31)

Such general considerations of the needs of children are matched by the actual pace of instruction by Japanese teachers. This is reflected in the number of pages of text covered in a period, which may range from one half to one page in the first grade, to from one to one and a half pages from grades two through six (MCLSG, 2009). This strikingly low rate is intended to ensure the understanding of all students, who are taught in mixed ability classrooms using whole class instruction. (Stevenson 2002, p. 104; Stevenson and Lee, 1997, p. 36).

But the single factor which best explains the high achievement and uniform progress of Japanese students is extremely effective teaching methods which are a product of the same classroom-based teacher research that has produced the focus and coherence of their curriculum. Lesson study, and the research groups that ensure the dissemination of its findings, began being carried out almost 140 years ago, from the establishment of the Japanese national education system. In his recently published book The History of Modern Japanese Education: Constructing the National School System, 1872-1890, Benjamin Duke recounts events that mark the beginning of activities which are the essence of present-day study groups and classroom lesson research. (Duke 2009, pp. 248-249). By the turn of the 20th century there were numerous journals disseminating the findings of lesson study (Sakar Arani et al 2010, p. 179, 181), and by 1930, the practice of lesson study had developed to the level at which it exists today (ibid., p. 186).

With so long a history of such involved professional activity, it is no surprise that the quality of Japanese teaching has reached such a high level. This results in the appearance that the curriculum is challenging or demanding because Japanese children are able to learn material that would indeed be so for our students. But if they began in first grade and had the same learning experiences, they could learn the same material as well as the Japanese. The illusion of rigor is simply an artifact of very effective teaching practices.

The Knowledge Base

Age appropriate subject matter, its optimum sequence, and the best methods to teach it are embodied in Japanese elementary math text books. Because they must conform to the Course of Study developed by the Ministry of Education, “…even though several companies produce textbooks in Japan, the books they produce are almost identical throughout the country.” (Stigler et al, 1996, p. 215). The aspects that are identical are the topics in each grade, the interconnection of topics between grades, and the activities and examples used to teach them. Textbooks from different publishers contain examples that have been found to be effective in developing an
understanding of a concept, and dispelling confusion that students often display in connection with it. Two examples occur in treating the subject of area.

Area is introduced formally for the first time in the fourth grade. In an instance of spending an entire period on a single problem (see Stigler 1988 above), students are asked “Which is larger, the square or the rectangle?”, after being given sheets containing a 4cm x 4cm square and a 3cm x 5cm rectangle. They are asked to think about how they could answer the question, and since they already know about perimeter, some try to measure that aspect of the figures to answer the question. They immediately observe that the perimeters are both 16cm and learn from this clear demonstration that in this case that quantity cannot be used to describe the size of a geometric figure. These dimensions were deliberately chosen, based on teachers’ past observations of children’s mistaken assumption that perimeter is an indication of the size of an object, and serve to eliminate this misconception. (Hironaka & Sugiyama 2000, 4B, p. 24; Kyôiku Shuppan 1993, pp. 32-33).

Another issue that has often been observed to be a source of confusion is the use of square units (e.g., the cm$^2$) to express the areas of nonrectangular shapes. Leaving nothing to chance, both texts give specific exercises in figuring out the areas of shapes other than squares or rectangles. (ibid., p. 25; ibid., p. 34; see Appendix B)

Specific features such as these and many others, from the carefully graded sequences of topics, to the interconnections between them in different grades, to concrete activities designed to foster interest and understanding in children, are all explained in the accompanying teachers manuals, and make up the knowledge base of Japanese elementary mathematics instruction.

**Conclusion**

Japanese math education is so effective because at the elementary level it operates on the assumption that learning something with understanding takes time and a consideration of children’s physical and intellectual needs. The very moderate pace of progress through the textbook and numerous activities involving concrete materials combine with intermittent periods of vigorous physical activity to produce optimal conditions for learning.

In addition to consistently meeting the basic needs of children, the teaching methods themselves are products of many years of classroom research by teachers. They have evolved into a carefully graded system of interconnected learning experiences which have coherence between, as well as within, the lessons, and become part of the textbooks, which are written by teams of experienced teachers. The development of concepts that may be addressed in detail at later levels is begun intuitively in very early grades. It extends throughout elementary school and into middle school, where the work in geometry gives the impression of such rigor.

But it is clearly not the mere choice to give children a “demanding” or “challenging” curriculum that develops their ability to do such work; it is the knowledge of ingenious methods to teach them effectively. Such methods, developed by experienced teachers who over many years, have produced a meticulously constructed system resulting in a high level of achievement in the average student. Instead of trying to make things more difficult by requiring more rigorous material for our students, we would do much better to prepare our teachers to use the techniques, materials, and the rational curriculum that are the products of Japanese elementary teachers’ classroom research.
References


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Appendix A

IEA stands for the “International Association for the Evaluation of Educational Achievement”, the real acronym for which ought to be IAEEA. IEA is actually an *abbreviation* for an *acronym*.

1. The very first IEA study, “The Pilot Twelve-Country Study”, was conducted in 1959-62.
2. The second IEA study, the “First International Mathematics Study” (FIMS) occurred in 1964.
3. The third study, conducted in 1970-71 was called the “Six Subject Study” and examined reading comprehension, science, literature, French, English and civics.
4. The fourth study was the “Second International Mathematics Study” (SIMS), conducted in 1980-81.
5. The fifth study was the “Second International Science Study” (SISS), the FISS being the science part of the “Six Subject Study”, carried out in 1970-71, which, as mentioned above, was the third IEA study. The SISS was carried out in 1983-84.

An examination of the “Brief History of the IEA” shows numerous other studies being conducted (addressing issues from preschool education, to literacy, and information technology in education) between 1984 and 1995, when the “Third International Mathematics Study was conducted. A new target population (9-10 year olds) was added in 1981, but the scores in mathematics for this population were not publicized till the 1995 study.

Links to IEA test history and scores on the mathematics and science tests are as follows:

International Association for the Evaluation of Educational Achievement
Brief History of IEA


TIMSS:
1995
[http://timss.bc.edu/timss1995i/TIMSSPDF/P1HiLite.pdf](http://timss.bc.edu/timss1995i/TIMSSPDF/P1HiLite.pdf)
[http://timss.bc.edu/timss1995i/TIMSSPDF/P2HiLite.pdf](http://timss.bc.edu/timss1995i/TIMSSPDF/P2HiLite.pdf)
1999
[http://timssandpirls.bc.edu/timss1999i/pdf/T99i_Math_All.pdf](http://timssandpirls.bc.edu/timss1999i/pdf/T99i_Math_All.pdf)
2003
2007
Appendix B

The contents of this appendix are intended to illustrate the uniformity in content and methods of teaching of the concept of area, over time and between publishers. Provided for this purpose are excerpts from 1) a chapter on area in a student text (shown in a teachers’ manual) for the fourth grade second semester in the series New Mathematics published by Kyôiku Shuppan, Inc. (1993) and 2) a student text from the series Mathematics for Elementary School 4B published by Tokyo Shoseki (2000).

Listed below are topics and related activities which appear in both texts in nearly identical forms, because they are part of the Japanese knowledge base for teaching about area for understanding:

- Tell which is larger, a similar sized square or rectangle to 1) develop the concept of area as a type of quantity and 2) motivate the use of units to express and compare the areas, introducing the square centimeter (cm²).
- “Disentangle” the ideas of perimeter and area as ways to express the size of a figure, because students frequently try to use perimeter to do so.
- Show that square units of area can be used to express the area of shapes that are not rectangular or square, which some students often believe is not possible.
- Develop the formulas for the area of a square and rectangle by using a series of diagrams which show square units placed along the base and height of the rectangle, and then completely filling it. After counting the squares, it becomes intuitively clear that it would be easier to multiply the number of columns by the number of rows (or vice versa). This then easily converts to base times height (or length times width).
- Apply the formula 1) to straightforward problems, 2) to problems with base and height of different units and 3) to problems with a known area and side to find the remaining side.
- Find the area of composite figures consisting of rectangles and squares, which are then expressed as math sentences or equations.
- Introduce each of the following units in its appropriate context (with activities for each one, if possible) and its relative size compared to other units previously learned: m², are (a square unit 10 m on a side), hectare (a square unit 100m on a side), and km².

The entire sequence spans ten class periods and forms numerous connections between area and previously learned concepts. These are—carefully moving from simple to more complex—using units to express the size of a quantity, multiplication (to facilitate counting), perimeter distinguished from area as an incorrect indicator of size, solving problems with a formula with an unknown quantity, and using equations to represent multistep calculations. These interconnections among concepts foster the thorough understanding that leads to the outstanding performance of Japanese students.
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2. The nature of the copyrighted work
3. The amount and substantiality of the portion used in relation to the copyrighted work as a whole
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FL-102, Revised May 2009
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GOAL OF THE FIRST SUB-UNIT

1. Understand the concept of area
2. Understand the unit of area
   (square centimeters)(cm²)

GOALS OF THE FIRST AND SECOND HOURS

- Understanding the concept of area
- Understanding the unit of area
  (square centimeters)(cm²)

Preparation

Tracing paper, Graph paper ruled into 1 cm squares

Example of Lesson Development

Leading Question 1

Which is larger, the square (x), or the rectangle (y)?

(Possible reactions)

a) x is larger
b) y is larger
c) both have the same area

Leading Question 2

Which answer is correct?

(Possible reactions)

a) Examine their perimeters.
b) Cut out the two figures and put one on top of the other.

HOW TO EXPRESS AREA (2 HOURS)

🌟 Let’s find out which figure is larger.
Is it the rectangle (x) or the square (y)?

(x)  (y)

◊ What kind of method could be used to figure this out?

On the guiding principles of this unit

We taught the concept of size and how to measure it in the first grade. At that time, we piled up paper or colored some grids on graph paper and counted the squares. In second grade we tried to cover the floor using congruent pieces of triangular or square tiles.

In this unit, the main goal consists of the understanding of area as an extensive quantity. Area is a quantity which is numerically expressed in certain standard units, such as cm², m², acre, hectare, km², so it is also required to explain these units in detail.

Because the concept of area as a bulk quantity is often quite vague, it is a good idea to familiarize students with this concept through practical experience. It is also important to make them familiar with formulae for calculating the areas of a square and a rectangle so that they will be able to combine these formulae when they have to calculate areas of more complicated shapes in the future.
Trace figures (x) and (y) onto a piece of paper. Cut out the figures, set them on top of one another, and see which is larger.

Cut out the parts that are sticking out and overlap them again.

Which one is larger, (x) or (y)? Answer: (y)

Put the figures (x) and (y) on graph paper ruled into 1 cm squares. Count the number of squares covered by each of the figures.

This concept is called area. Area is expressed in square units.

**Guidance**
It should be emphasized that the figures can not be compared in terms of the lengths of their perimeters. Make students aware of the fact that the figures can be compared by overlapping cut-out figures and seeing which one sticks out more.

**Leading Question 3**
Ask questions қ, қ, and қ

**Guidance**
Identify that the square (y) has a greater area than the rectangle (x).

Since we will compare the parts of the figures that stick out when they are overlapped, it is easier to use different colored pieces of paper to represent each figure.

**Leading Question 4**
Ask question қ

[Possible reactions]
(x) = 15 units  (y) = 16 units

**Guidance**
At this point, we do not encourage students to use the formula for area of a rectangle, but we would like students to count the number of small squares, whose sides are 1 cm long, within the specified figure.

Inform students that the concept of area can be expressed in terms of square units. Also tell them that the word “area” will be used for this concept.

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**How to pronounce [square centimeter]**

In daily conversation, cm and cm² are said [centi-] and [square centi-], respectively.

Note from editor: This applies to Japanese terms and pronunciation only.

The word [centi-], however, simply means “one-hundredth”. As long as we are discussing mathematics in our class, we would like to say “centimeter” and “square centimeter”, respectively.
**Guidance**

Introduce the unit \( \text{cm}^2 \) that is used to numerically express an area.

Let the students get the feeling of the size of the unit \( \text{cm}^2 \), and let them practice the way of writing and reading the unit.

**Leading Question 5**

Ask the question *What is the area of the figures (x) and (y), respectively?*

**Possible reactions**

(\( x = 15 \text{ cm}^2 \))  (\( y = 16 \text{ cm}^2 \))

**Guidance**

Help students grasp the fact that they only have to tell how many unit squares are within the figures in question, as was done in Question 4.

Make sure that they realize that the figure (y) is larger than (x) by 1 \( \text{cm}^2 \).

[1] If you use half-sized rectangular or right triangular tiles, you can also show areas of 1 \( \text{cm}^2 \) with different shapes.

Show the students clearly that (y) and (z) also have areas of 1 \( \text{cm}^2 \).

[2] When a variety of new figures are created, use an overhead projector to show possible variations. If there are students who cannot think about anything other than a combination of squares and rectangles, show them examples like (y) and (z) in [1] and encourage them to think more creatively.

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**On the unit square**

Define the area of a square whose sides are 1 cm long as 1 \( \text{cm}^2 \). This square is called a unit square, because the area of this square is used as a unit of measurement when we measure the unit of an arbitrary figure.

As we see in the examples (y) and (z) of [1], halves of the unit square cut along the diagonal or along the line connecting the midpoints of the opposite sides can be reassembled to have the same 1 \( \text{cm}^2 \) area.
AREA OF SQUARES AND RECTANGLES
(2 HOURS)

Let's consider how to find the area of the square and rectangle shown below.

How many unit-squares can be put in vertically?
Answer: 4

How many unit-squares can be put in horizontally?
Answer: 6

Let's work out how to calculate the area of a rectangle.

- 4 unit-squares are inserted vertically
- 6 unit-squares are inserted horizontally,
  thus the area is 24 cm².

\[ 4 \times 6 = 24 \]
(vertial) (horizontal) (area)

On the formulae

Formulae are generally expressions of relationships between several quantities where the expression is known to be valid irrespective of the magnitudes of these individual constituents. It is expressed in the form of an equation. For example:

(Area of rectangle) = (Length of base) \times (Length of height)

We emphasize that the importance of the formulation lies in:
1) Derivation of the formulae
2) Application of the formulae

GOAL OF THE SECOND SUB-UNIT
1. To understand how to find the area of rectangles and squares and the formulae of these areas
2. To improve students' ability to solve problems using the formulae of the areas of geometric figures

GOAL OF THE FIRST PERIOD
• Understanding how to find the area of squares and rectangles and the formulae for these operations

Preparation
Graph paper ruled into 1 cm squares, rulers

Example of Lesson Development

Leading Question 1
Show students a rectangle with sides whose lengths are not indicated. Then ask: "How can we measure the area of this rectangle?"

[Possible reactions]
a) Place as many 1 cm² tiles on the rectangle as you need to cover the area and count the number of tiles used.
b) By visual observation, you can see that 4 squares of 1 cm² will be placed vertically and there will be 6 rows of squares. Thus, 4 \times 6 = 24 (cm²)
c) By visual observation, six squares of 1 cm² will be placed horizontally and there will be 4 rows of squares. Thus, 6 \times 4 = 24 (cm²)

Guidance
First, mention that method a) is fundamentally correct. Then explain that both b) and c) are equally efficient ways.

Leading Question 2
Ask question.
(Reactions b and c)

Guidance
The area of a rectangle is equal to the product of the number of vertical unit squares and the number of horizontal unit squares. This should be made very clear.
Leading Question 3

Ask 🌟

Guidance
Area of a square can also be calculated as the product of the lengths of the base and height. Stress that, in this case, the lengths of the base and the height are equal.

Leading Question 4
“Let’s describe, in words, how to get the value of the areas of the square and rectangle”

Guidance
From 🌟 and 🌟, the areas of the rectangle and the square are given by
(height) x (base)
(side) x (side), respectively.

Make sure the students understand the expressions, and tell them that we call these “formulae”.

Question 5
Ask 🌟

Guidance
Combine the formulae for the areas of the rectangle and of the square, case by case, and let the students calculate the differences of the areas of a square and a rectangle.

[1] First, apply the formula to calculate the area of the square or the rectangle, and then divide the area into unit squares to verify the calculation.

Let’s calculate the area of the square on the previous page.

$$5 \times 5 = 25$$

(Side) (Side) (Area)

Let’s summarize what we know about the area of a rectangle and of a square.

Area of a rectangle = height x base
Area of a square = side x side

These are called “Formulae for the area of the rectangle and of the square”.

Which area of the rectangle or of the square shown on the previous page is larger? By how much?

Answer: 25 - 24 = 1
The area of the square is larger by 1 cm².

[1] Find the area of the following figures.

1. Rectangle that has the height of 9 cm and the base of 8 cm.
   $$9 \times 8 = 72$$
   Answer: 72 cm²

2. Square whose sides are 7 cm.
   $$7 \times 7 = 49$$
   Answer: 49 cm²

A note on guidance for derivation of the formula

When you derive the formulae of areas of the rectangle, do not use just one example, but use a few examples first, and then describe what they have in common in words. It is also important to make sure that the areas of a rectangle of any shape can be calculated, once we know the lengths of the base and height.
How many times is the area of (x) greater than the area of (y)?

(x) 36 x 24 = 864
(y) 18 x 24 = 432
864 \div 432 = 2
(36) \div (18) = 2
Answer: 2

We would like to draw a rectangle whose area is 48 cm² and whose base is 6 cm.
What would be the length of the height of this rectangle?

Let the length of this rectangle be cm and try to write down the formula of the area:
48 = x 6

Find the number that satisfies the above formula.
48 \div 6 = 8

What is the length of the base of a rectangle whose area is 108 cm² and whose height is 9 cm?

9 x cm = 108
108 + 9 = 12
Answer: 12 cm

On the equation involving the symbol □

When you are setting up the equation involving □, e.g. □, you should not simply write 48 \div 6 = □, but instead you should first explain the meaning of the formula, i.e. 48 = □ x 6, and then solve the equation. By reminding students how the formula is derived, (i.e., If we let a be the height, b be the base, and c be the area, then we have a \times b = c), then we can see that knowing two of the three quantities, a and c, say, then we can find the value of b; that is to say, we can instruct the students in an algebraic understanding of the equation.

First, let the students examine the drawings. Then, using the formula for area, calculate the areas.

twice (36) x 24 (x)
(18) x 24 (y)
and see (x) is really twice larger than (y)

GOAL OF SECOND HOUR

To understand the way to find the length of a side using the formula for area

Preparation
Flash cards of mathematical formulae, ruled blackboard

Example of Lesson Development

Leading Question 1
(Ask the students to close their textbooks, and write down on the blackboard what is asked.)

Express the quantity to be found as □. Draw in your notebook what the question * means.

Guidance
If the diagram in the textbook can be drawn, it is just as satisfactory.

Leading Question 2
Ask □ and □.

Guidance
Set up an equation referring to the formula.

[3] It would be good to ask students to draw the diagram which represents the equation before they set up the equation. It would also be good to have students substitute their answers back into the original equation to check their validity.
GOAL OF THE THIRD HOUR

- To understand how to calculate areas of some complicated shapes

Preparation

Ruled blackboard, print-outs from textbook

Example of Lesson Development

Leading Question 1

"Find the area of the figure on the right using as many methods as you like"

Guidance

Let students think independently, and let them solve the problem by themselves.

Leading Question 2

Ask

Guidance

For the figures, a), b), and c), make it clear that the total areas would not change, even if the figures were cut along the dotted lines, or rejoined to make their original shapes.

Leading Question 3

Ask

Guidance

Following the sequence (a), (b), (c), calculate the area. After calculating the area, discuss the good and bad points of the students' calculations.

[4] Make sure the students notice that the area of a field is independent of the location of the road. Then let them figure out the way to calculate the area of the field.

Supplemental Problems

1. Find the areas of the following figures.
Let’s think about a way to compare and express the size of these shapes!
1. As shown below, divide the sides of the rectangle and the square into \(1\text{ cm}\) segments and compare how much space they each take up.

![Diagram of a rectangle and a square divided into \(1\text{ cm}\) segments]

2. How many \(1\text{ cm}\) squares are there inside the rectangle and the square?

3. The amount of space inside the rectangle and the square can be expressed by the number of \(1\text{ cm}\) squares that fill up the space.

The amount of space inside a shape is called the **area**.

The area of a square with \(1\text{ cm}\) sides is called **square centimeter**, and it is written as \(1\text{ cm}^2\).

Square centimeter is a unit used to express area.

4. How many square centimeters are the areas of the rectangle and the square on page 23?
1. How many square centimeters is the area of each shaded part below?

2. Draw many shapes with an area of $4 \text{cm}^2$.

**Math Story**

**Areas of Shapes with Equal Perimeters**

All the shapes below have equal perimeters, but their areas are different.
**Area of Rectangles and Squares**

Formula for the area of rectangles and squares

1. Let's think about a way to calculate the area of a rectangle or a square!

1. How many \(1\,cm^2\) squares can be lined up along the length (vertical side) of the rectangle? How many can be lined up along its width (horizontal side)?

2. How many \(1\,cm^2\) squares are there inside the whole rectangle? Let's multiply to find out! What is the area of the rectangle?

3. How many \(cm^2\) is the area of the square? Let's multiply to find out!

!! In order to calculate the area of a rectangle or a square, follow these steps:

1. Measure the lengths of two sides that are next to each other.
2. Multiply the two numbers that represent the lengths of the two sides.
Area of a rectangle = length \times width
Area of a square = side \times side

The above math sentences are called **formulas** for the area of a rectangle and a square.

1. How many $cm^2$ are the areas of the rectangle and the square below?
   (1) A rectangle that is 15cm long and 23cm wide.
   (2) A square that is 20cm on each side.

2. Measure the sides of the rectangle on the right and find the area.

3. Find the area of a rectangle that is 30mm long and 6cm wide.

When calculating area, you need to use the same units of length for all the sides.
4. In order to draw a rectangle with an area of $28 \text{ cm}^2$, that is 7 cm wide, how many cm long should it be?

Finding the area of composite figures

2. Let's think of a way to find the area of the shape on the right!

1. Let's explain each friend's way of thinking!

   Naoko  
   Kazuya  
   Sayuri

2. Let's find the area using each method!

1. How many $\text{ cm}^2$ is the area of the shape on the right?
Exercise 1

1. How many $cm^2$ is the area of shaded part?

2. Using a 20 cm long wire, make a rectangle that is 3 cm long. How many $cm^2$ is the area?

Challenge

Which Math Sentence?

Which figure below corresponds best to each of the following math sentences?

1. $10 \times 4 + 7 \times 5 + 10 \times 2$
2. $3 \times 4 + 7 \times 9 + 10 \times 2$
3. $10 \times 11 - 3 \times 5$
4. $3 \times 4 + 7 \times 11 + 3 \times 2$

A  
B  
C  
D