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School Geometry in America and China:

Divergent Paths, Contrasting Results

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Abstract

PISA and TIMSS results show that geometry has consistently ranked last among mathematics content domains in the United States, in sharp contrast to East Asia. This article examines the divergent histories of geometry education in the United States and China and explores three major debates: deductive reasoning versus intuition and practical applications; triangle similarity versus congruence as the principal vehicle for proof; and traditional side-angle versus transformation-based criteria for congruence and similarity. The analysis concludes that the erosion of training in logical reasoning through successive curriculum reforms is the root cause for American students' poor performance in geometry, while China has largely preserved the Euclidean tradition. The article emphasizes the importance of maintaining the rigor and logical structure of Euclidean geometry, while allowing flexibility for students of different abilities.

Keywords: Geometry education; Euclidean geometry; Curriculum reform; Deductive reasoning; PISA and TIMSS

1. PISA & TIMSS: Geometry Ranks Lowest among Mathematics Content Domains in the West

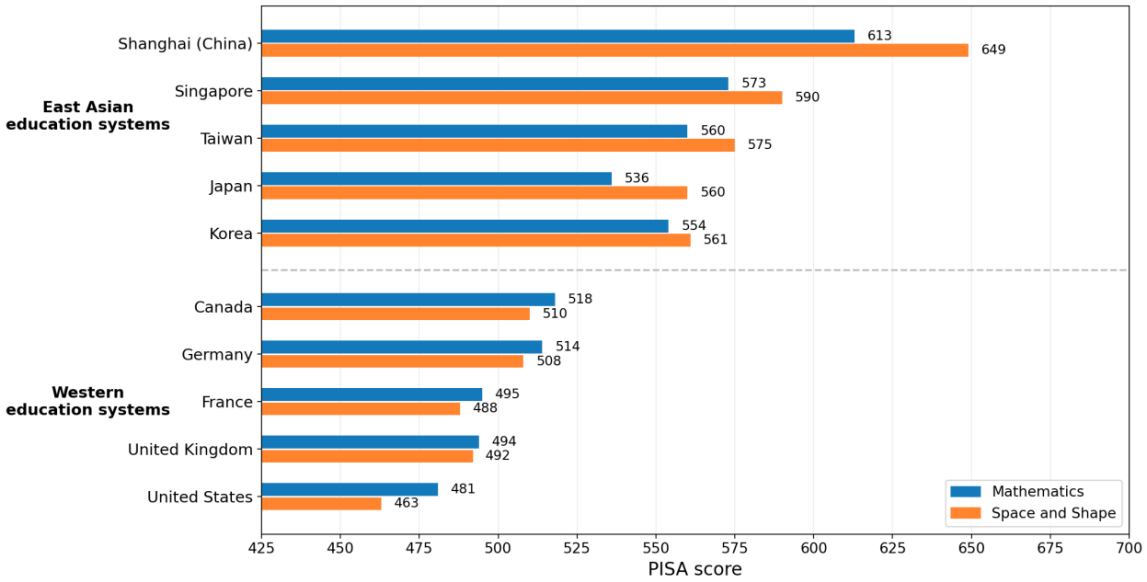
PISA 2025 results were released in September 2026. As in previous rounds of PISA and TIMSS, East Asian education systems substantially outperform Western countries in both mathematics and science, with a striking gap between the two groups. The four participating Chinese regions (B-S-J-Z) ranked first in both, clearly ahead of Singapore. China, Singapore and America scored 612, 563 and 463 in mathematics, respectively, with China outperforming the United States by nearly 150 points.[1]

B-S-J-Z—Beijing, Shanghai, Jiangsu, and Zhejiang—are among the most developed regions of China. What if the United States also sent only its highest-performing states to PISA? The recently published *2026 American Talent Report* ranks Massachusetts—the nation's top-performing state—only 20th globally in mathematics.[2] At the same time, B-S-J-Z's combined population of nearly 200 million lends weight to the comparison.

The purpose here is not to rank countries or education systems. Rather, the comparison raises a more fundamental question: What happens when different curriculum structures and teaching traditions operate over many years?

In America's century-long "math wars," [3] geometry has been the most extensively reformed—and most fiercely contested—area of mathematics education. What have these reforms achieved? PISA 2012 provides relatively comprehensive data. Figure 1 compares overall mathematics scores with scores in Space and Shape (geometry-related content), across East Asian and Western countries in that year. The contrast is striking: in East Asia, scores in Space and Shape generally exceed overall mathematics scores, whereas the reverse is true in Western countries, with the United States showing the largest gap.[4]

Figure 1. PISA 2012: Mathematics and Space & Shape Scores in Selected East Asian and Western Systems



Source: OECD, PISA 2012 Results (Volume I), Figure I.2.52.
Note: Shanghai (China) refers to the city participating in PISA 2012, not China's national average.

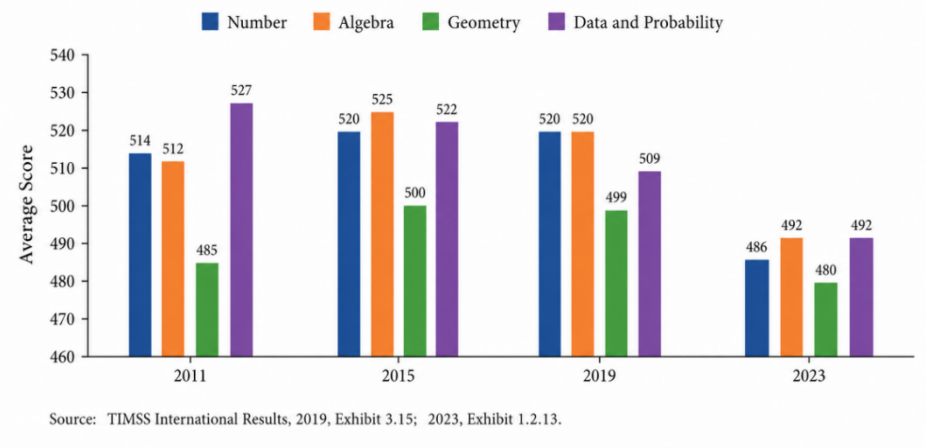
(Note: in Figures 1 and 2, the horizontal or vertical axes do not start at zero, allowing score differences to be shown more clearly.)

PISA 2012 found that seven of the twelve mathematics items on which American students performed particularly poorly belonged to "Space and Shape," while none of their twelve relative strengths fell in that category. Item-level comparisons also revealed substantial gaps between American and Shanghai students in geometric reasoning and problem-solving.

TIMSS results reveal a similar pattern. As Figure 2 shows, Geometry and Measurement (green) consistently ranked last among the mathematics content domains for U.S. eighth-graders, across all four assessment cycles from 2011 to 2023.

PISA and TIMSS have repeatedly sounded the alarm for Western education. Their wealth of data, particularly comparisons across content domains, offers valuable insights into the weaknesses, their underlying causes, and possible paths to improvement. These findings urge Western policymakers and educators to learn from their own history and study the experience of China and other East Asian systems.

Figure 2. U.S. Grade 8 TIMSS Mathematics Scores by Content Domain (2011-2023)



(Note: in Figures 1 and 2, the horizontal or vertical axes do not start at zero, allowing score differences to be shown more clearly.)

The author has long studied reforms in mathematics and science curricula and their consequences, publishing several articles in Chinese and English. [5–9] One of these, "Integrated Curriculum Reform and its Impact on Science Education," argues that physics has suffered the most from science curriculum reforms. Similarly, this article traces the divergent paths of school geometry in the United States and China, examines the major differences between the two curricular systems, and seeks to explain why geometry has become the most challenging area of mathematics for American students.

2. How Did Geometry Curricula Diverge in America and China?

In the 1990s, while teaching mathematics in British Columbia, Canada, I found that school geometry focused largely on recognizing figures and calculating lengths and areas. Logical reasoning and proof were limited, emphasizing triangle congruence, with little attention to similarity.

I was astonished. In my secondary-school years in China in the 1960s, congruence received little attention; we worked through demanding but fascinating proofs involving similar triangles. I regard that experience as my most valuable early mathematical training, profoundly influencing my later studies in mathematics and physics, and my way of thinking.

From that point on, the striking differences between Chinese and North American school geometry remained in my mind. Only later, as I traced their history, did I realize that the West

had already begun reshaping traditional geometry, and China and the United States gradually took divergent paths.

During the Cold War, their separation into opposing blocs drove them even further apart. Through successive reforms, the United States gradually weakened deductive proof in geometry, shifting toward intuition, practicality, and transformations. In China, although the Cultural Revolution severely disrupted education for a decade, subject-based education was rapidly restored after reform and opening up, and mathematics education retained the Euclidean geometry tradition.

When the two countries reengaged in the 1980s, China initially sought to follow Western reforms but soon recognized their problems and returned to its earlier course. (For a detailed historical account, see Appendix 1.)

Besides China, East Asian education systems, including Singapore, Japan and Taiwan, have largely retained the traditional Euclidean framework too. [10] Today, the consequences of two different approaches are visible in the enormous gaps in PISA and TIMSS performance.

To compare the geometry knowledge of students in the West and East Asia, the author compiled a table of percent correct for U.S. and East Asian students using data from the TIMSS 2019 eighth-grade mathematics assessment (see Appendix 2). The results reveal several patterns:

U.S. students performed relatively well on basic Cartesian coordinate questions, though they still lagged behind their East Asian counterparts. About one-third correctly answered basic questions on angles, lengths, and areas. However, they performed much worse on problems requiring the integrated application of geometric concepts and more than one step of reasoning. On problems involving similar triangles, the Pythagorean theorem, and area relationships, only about 20% or fewer earned full credit. In addition, geometric transformations such as rotations and reflections proved challenging for students in both regions.

There has been no lack of opposition within the United States. For generations, mathematicians, educators, and parents have criticized reforms that weakened mathematics education,[11] but have failed to reverse the trend. Today, criticism of the Common Core State Standards is growing. The *Archimedes Standards*, advocating a return to traditional mathematics education, have emerged;[12] and some schools are working to restore Euclidean geometry. Yet

decades of accumulated effects are difficult to reverse. Rebuilding rigorous, systematic mathematics education will likely be a long process.

China, meanwhile, owes much to distinguished mathematicians and educators such as Jiang Boju[13] and Zhang Dianzhou[14]. Their courageous public criticism, together with opposition from frontline educators and teachers, prompted a reassessment of the reforms and pulled school geometry back from the brink of abandoning rigorous reasoning. Without their efforts, a decline in educational quality might have been inevitable, jeopardizing China's ability to cultivate talent in STEM and AI fields.

3. Three Major Debates over the Geometry Curriculum

The differences between the two geometry curricula center on three fundamental questions:

- Should geometry emphasize logical reasoning or intuition and practical applications?
- Should triangle congruence or similarity serve as the principal vehicle for proof?
- Should congruence and similarity be established through side-angle relationships, or through transformations?

The following discussion addresses these questions.

3.1 Rigorous Logical Reasoning or Intuition and Practicality?

Mathematics is a discipline founded on rigorous logic; without that, it ceases to be mathematics. Euclidean geometry, deriving theorems from axioms through deduction, has long been a model of mathematical rigor. It was one of the enduring flowers produced by the “Greek miracle.” From Copernicus, Galileo, Descartes, and Newton to Einstein, the deductive rationality embodied in Euclidean geometry has remained one of the important intellectual sources of mathematics and science.

Yet today, even as mathematics itself continues to advance, mathematics education has progressively weakened the deductive structure of Euclidean geometry and the training in proof that accompanies it. This is certainly a step backward.

The value of Euclidean geometry goes far beyond learning the properties of geometric figures. It offers students a rare opportunity to develop logical reasoning through systematic training. Replacing rigorous proof with intuitive guesswork inevitably diminishes this vital role of mathematics education.

Research in mathematics education has identified essential elements of this training: students must look beyond visual appearances and understand geometric figures through definitions and theorems; must recognize the logical relationships among assumptions, propositions, and conclusions, and construct coherent chains of reasoning; and also understand why deductive proof establishes universal truths rather than merely confirming particular examples. Geometric proof thus cultivates a way of thinking that moves from visual intuition to abstract concepts, and from empirical observation to logical necessity. [15]

The author experienced this firsthand. Algebra and trigonometry certainly involve logical reasoning, but rarely with the same degree of rigor, coherence, and systematic development. I believe this experience is shared by generations of Chinese scientists and engineers.

The ability to reason through extended chains of deduction is fundamental to advanced mathematics, theoretical physics, algorithm development, and other demanding fields. This sustained mathematical preparation has been an important strength in China's cultivation of scientists and engineers.

Over the past century, successive reforms have gradually weakened Euclidean deductive proof,[16] and even the name “geometry” was replaced by “Space and Shape.” Visual intuition and practical applications certainly have their place in geometry teaching. But abstract logical reasoning lies at the heart and must not be weakened.

3.2 The Main Vehicle for Geometric Proof: Triangle Congruence or Similarity?

Triangle similarity is far from an abstract concept detached from reality. It allows us to measure heights and distances using shadows and lines of sight, to draw maps, to study optical imaging, and to create architectural plans and engineering models to scale. Its central principle—preserving shape while changing size—enables us to infer the unknown from the measurable and understand large structures through small models. Similarity captures the relationship between shape and scale in the physical world, serving as a bridge between geometry and reality.

Reasoning and proof in Euclidean geometry are built upon the definitions and properties of points, lines, planes, polygons, triangles, and circles. A triangle is the simplest stable polygon, and any polygon can be divided into triangles. Even intersecting chords in a circle give rise to pairs of similar triangles. It is therefore natural that triangles serve as fundamental objects of study in plane geometry, with similarity playing a central role.

The criteria for triangle similarity are relatively simple: two pairs of equal corresponding angles (AA); or one pair of equal corresponding angles and two pairs of proportional adjacent sides (SAS); or all three pairs of proportional corresponding sides (SSS). Any one of them guarantees the same shape. They provide a simple and practical theoretical basis for the applications discussed above and lay the foundation for trigonometry.

Triangle similarity introduces proportional relationships, bringing together shapes and numbers and enriching geometric proofs. Therefore, similarity plays a role that congruence cannot replace. American geometry curricula have shifted their emphasis toward congruence, thereby reducing opportunities for students to develop complex deductive reasoning skills. On the two TIMSS 2019 items involving similar triangles, only one in five American students received full credit, a result closely associated with the reduced emphasis on similarity.

The two issues discussed above both concern curriculum adjustments in response to the expansion of mass education. Admittedly, demanding proofs involving triangle similarity are suitable only for some students; for others, they may impose excessive cognitive burdens without corresponding benefits. Even for highly capable students, greater difficulty and depth are not always better, and excessive drilling in clever tricks should be kept in check. Some geometry problems in Chinese middle schools today even involve optimization, reaching levels that might challenge mathematics professors.

While advanced proofs need not be required of every student, the fundamentals of Euclidean geometry are both essential and accessible to all. Using side-angle relationships to establish triangle congruence and similarity is intuitive and easy to grasp. The basics of inductive and deductive reasoning and logical proof, such as two-column proofs, should also be taught to all students.

Differentiated instruction is therefore needed. Students pursuing advanced studies may benefit from challenging deductive proofs and some higher-level mathematics, while others could focus on visual understanding, practical applications, and useful skills. The goal is to preserve rigorous logical training for future scientists and engineers while freeing other students from being forced into the same difficult drills.

American K–12 education already offers differentiated courses and learning pathways, while China's more uniform curriculum tends toward a one-size-fits-all approach. Yet Professor

Zhang Dianzhou advocated differentiated instruction long ago. It is time to put Confucius's principle of teaching according to individual abilities into practice.

3.3 Congruence and Similarity: Side-Angle Criteria or Transformations?

Another major difference lies in how congruence and similarity are established.

Congruent figures have the same shape and size. Triangle congruence is also determined by three criteria: ASA, SAS, and SSS. Here, S means equal rather than proportional sides. The original meaning of triangle congruence and similarity lies in the equality of three pairs of corresponding angles, with corresponding sides either equal or proportional. Euclidean criteria, however, establish all six relationships from just two or three side-angle conditions, demonstrating the elegance and power of Euclid's axiomatic deductive system. Moreover, these criteria require no tools such as coordinates or vectors.

Transformation geometry defines congruence through rigid motions—translations, rotations, and reflections—that bring two figures into exact coincidence. Similarity is defined through dilations combined with rigid motions. These are primarily definitions rather than directly applicable criteria.

While transformations seem intuitive, visualizing geometric motions is quite different from defining them rigorously as mathematical operations. The latter involves more abstract concepts such as coordinates, vectors, and mappings, thereby increasing the theoretical demands on students.

The transformation-based criteria arose mainly from the extension of modern mathematical ideas into school curricula, not from any shortcomings of Euclidean criteria in teaching. During the 20th century, concepts such as set theory and abstract algebra entered school mathematics, provoking a backlash among mathematicians and educators. French mathematician René Thom mocked this excessive abstraction, divorced from intuition: "If a chicken is in a coop, and the coop is on a farm, does a fox need set theory to know that the chicken is on the farm?"[17]

In contrast, many Chinese mathematicians and education critics have argued that beginning geometry with transformations of figures takes an unnecessary detour. They favor the traditional Euclidean approach, grounded in concrete side-angle relationships. Chinese textbooks introduce translations, rotations, and reflections as intuitive tools, but return to classical side-angle and proportional reasoning when teaching congruence and similarity.

Geometry began over two thousand years ago with the simple yet rigorous Euclidean system of ancient Greece. Descartes later introduced Cartesian coordinates and founded analytic geometry. The emergence of non-Euclidean geometries in the nineteenth century challenged its uniqueness. Klein then brought different geometries under a unified framework. Thus, geometry evolved from the study of shapes and their side-angle relationships to coordinates and algebra, and eventually to transformations, structures, and the abstractions of modern mathematics.

Mathematical knowledge has developed through these successive stages. Should students, then, skip the earlier stages and begin directly with the abstract frameworks of modern mathematics?

The historical-genetic principle in education suggests that individual cognitive development, to some extent, retraces the historical development of scientific knowledge. The progression from simple to complex, concrete to abstract, and elementary to advanced often provides a natural path for learning. It allows new abstractions to build on existing understanding while giving students opportunities to experience the intellectual breakthroughs that marked the history of human knowledge.

I remember experiencing such a moment when I moved from Euclidean to analytic geometry. It felt like opening a new window: geometry and algebra could work together! My familiarity with geometric figures, side-angle relationships, and proofs enabled me to appreciate the intellectual leap that analytic geometry represented. That sense of discovery is itself an important part of learning.

Later generations need not retrace every step in the history, but they cannot simply skip essential stages of cognitive development or leap from ancient to modern mathematics. Replacing traditional side-angle criteria with transformation geometry in middle school violates this principle. What appears sophisticated is, in fact, an attempt to force intellectual development prematurely. This is another serious misconception underlying modern Western geometry education reform.

4. Lessons from History

Although the disruption of education during China's Cultural Revolution was drastic, it was reversed after a decade. By contrast, the weakening of academic foundations in American education has been gradual and persistent, spanning six to seven decades, with far more profound

cumulative consequences.

As discussed earlier, the expansion of mass education is a driving force in the evolution of Western geometry curricula. Modern education systems face a persistent dilemma: how to balance elite education with mass education. Broadly speaking, the West sacrificed talent by lowering the bar, while China wore down ordinary students by raising it.

The United States has not neglected gifted education. Differentiated courses and ability grouping have long been Western traditions. The real problem lies in successive curriculum reforms, whose sweeping changes in content and teaching approaches have undermined the very foundations of education.

Despite fluctuations, reforms have generally moved toward lower academic standards to accommodate average students. Even accelerated pathways cannot compensate for mathematics and science curricula that lack the rigor and depth needed for advanced study. A small proportion of elite schools retain greater curricular freedom, but they too face the pressures of nationwide reforms. Meanwhile, gifted students in ordinary public schools may receive only superficial knowledge, leaving their potential undeveloped.

America's shortage of homegrown scientific and engineering talent, and its heavy reliance on immigrants, are rooted in significant weaknesses in its K–12 education.

China faces the opposite problem. Its strong commitment to education has extended elite academic standards to nearly all students, subjecting the majority to demanding training designed for a minority. The belief that prestigious universities offer upward social mobility, reinforced by decades of higher education expansion, has fueled an intense pursuit of degrees. China now produces university graduates, master's degree holders, and PhDs in numbers that may exceed labor market demand.

As degrees become more common, their job-market value declines, and the pursuit of credentials begins to subside. Ordinary families increasingly turn toward practical skills and career opportunities. Hopefully, this shift will foster a healthier system of educational pathways suited to different students.

In short, the American experience shows that expanding educational opportunities should not come at the expense of academic standards. China, meanwhile, demonstrates that maintaining academic rigor and depth need not mean imposing uniform expectations on all students.

The different paths of geometry education in China and America offer a valuable lesson. After decades of practice, reforms once hailed as "advanced" can now be evaluated in light of their actual results. Educational reform shapes generations of students. It must therefore respect the principles of each discipline and cognitive development and proceed with caution.

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References

1. OECD. *PISA 2025 Results (Volume I): Future-Ready Students*. Paris: OECD Publishing, 2026. https://doi.org/10.1787/73451bc5-en.
2. Johnston, Emerson, and Amy Zegart. *The 2026 US Talent Report: Benchmarking America's STEM Pipeline Amid the Great Talent Competition*. Stanford Institute for Human-Centered Artificial Intelligence and Hoover Institution, Stanford University, September 28, 2026.
3. Lianxi, "The Century-Long Math Wars in the United States", *Pipixia WeChat Official Account*, 2017 (in Chinese).
4. OECD. (2013). Lessons from PISA 2012 for the United States. Strong Performers and Successful Reformers in Education. OECD Publishing, Paris, Tables 3.2–3.4. https://doi.org/10.1787/9789264207585-en
5. Shen, Qianruo. "The Discovery-Based Curriculum Undermining Elementary Mathematics." *Bulletin des Sciences Mathematics* 56, no. 11 (2017): 7–11. (in Chinese)
6. Shen, Qianruo. "Curriculum Reforms That Have Seriously Undermined STEM Education in the United States and Canada." *Physics and Engineering,* 2019, 29(5): 27–33. (in Chinese)
7. Shen, Qianruo. "Integrated Curriculum Reform and Its Impact on Science Education: Why Is the West Falling Behind East Asia in PISA and TIMSS?" *Nonpartisan Education Review*, 2024, 20(1): 1–14.
8. Shen, Qianruo. "Visual Models, Procedural Fluency, and Multiple Strategies: A Cognitive Critique of Common Core Elementary Mathematics." *Nonpartisan Education Review*, 2026, 22(1): 1–11.
9. Shen, Qianruo. "Rethinking the Structure of American Secondary Education." *Nonpartisan Education Review*, 2026, 22(1): 1–11.
10. Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan. Course of Study for Lower Secondary Schools: Mathematics. 2017; Ministry of Education, Singapore. Mathematics Syllabuses: Secondary One to Four. 2020; Ministry of Education, Taiwan. Curriculum Guidelines of 12-Year Basic Education: Mathematics. 2018 (implemented in 2019).
11. Klein, David, Richard Askey, R. James Milgram, Hung-Hsi Wu, Martin Scharlemann, and Betty Tsang. "An Open Letter to United States Secretary of Education, Richard Riley." *The Washington Post*, November 18, 1999.
12. National Association of Scholars, & Freedom in Education. *The Archimedes Standards: Model PreK–12 State Mathematics Standards*. New York, NY: National Association of Scholars, 2025.
13. Jiang, Boju. "What Has the New Curriculum Standard Taken Away from Mathematics Education?" *Guangming Daily, Education Weekly*, March 16, 2005, p. 5. (in Chinese)
14. Zhang, Dianzhou. "A Review and Prospects of Plane Geometry Teaching." *Mathematics Teaching*, 2005, No. 5: 2–7. (in Chinese)
15. Fujita, T., Kondo, Y., Kumakura, H., Miyawaki, S., Kunimune, S., & Shojima, K. (2026). *A proposal of the competency-based teaching sequence for deductive proofs in geometry in secondary schools. Asian Journal for Mathematics Education, 5*(3), 391-414.
16. Klein, David. *A Brief History of American K-12 Mathematics Education in the 20th Century.* In Mathematical Cognition, edited by James Royer, 175–225. Greenwich, CT: Information Age Publishing, 2003.
17. Thom, René. *Modern Mathematics: An Educational and Philosophical Error? American Scientist* 59, no. 6 (1971): 695–699.

Appendix 1: How School Geometry Diverged in America and China

School Geometry in America and China:
Divergent Paths, Contrasting Results

A Century of Change in the American School Geometry Curriculum

- **Traditional geometry** followed the Euclidean framework, emphasizing deductive proofs based on axioms and definitions, with triangle similarity as the principal vehicle.
- **Late 19th to early 20th century:** Felix Klein proposed a unified view of geometry through transformation groups and their invariants, while David Hilbert reconstructed Euclidean geometry through modern axiomatization.[14,15] These ideas gradually influenced school geometry. Progressive education later challenged traditional geometry as overly abstract, introducing intuitive and experimental approaches, though the Euclidean framework remained dominant.
- **1950s–1980s:** Driven by Cold War competition, the *New Math* movement introduced set theory, abstract algebra, and geometric transformations into school mathematics. In the following decades, criticism of excessive abstraction prompted a return to basics, with Van Hiele's theory of geometric thinking gaining influence in geometry education.[16] Yet triangle congruence was increasingly emphasized over similarity as the main vehicle for proof, and deductive reasoning continued to decline as secondary education expanded.
- **1989–2000:** The standards of the National Council of Teachers of Mathematics (NCTM) placed greater emphasis on transformations, student exploration, and real-world applications, while reducing the role of deductive proofs.[17] Controversy continued. In 1999, David Klein and more than 200 scholars signed an open letter opposing several reform mathematics programs recommended by the U.S. Department of Education.[18]
- **2010:** The *Common Core State Standards* established transformations as the basis for defining congruence and similarity.[19] After more than a century of reform, American school geometry had developed a curriculum markedly different from the traditional Euclidean model.

The Preservation of Euclidean Geometry in Chinese Mathematics Education

- **Early 20th century:** Euclidean geometry entered Chinese schools with the establishment of modern education, fostering a tradition emphasizing foundational knowledge and deductive proof. After 1949, Soviet influence further strengthened systematic geometry instruction. Subsequent curriculum and textbook revisions placed greater emphasis on the "Two Basics"—basic knowledge and basic skills. Until the outbreak of the Cultural Revolution, the Euclidean deductive framework remained a cornerstone of school geometry.
- **1966–1976:** The Cultural Revolution brought normal schooling to a halt for several years and severely disrupted education for a decade. In the name of "popularization" of education, some physics, chemistry, and biology content was merged into courses called "Fundamentals of Industry" and "Fundamentals of Agriculture." Traditional disciplinary structures were largely dismantled, leaving an entire generation with severe deficiencies in basic knowledge of science and mathematics.
- **1978–2000:** Following the launch of Reform and Opening Up and the restoration of educational order, basic education recovered rapidly. Despite repeated adjustments, the deductive framework of plane geometry was preserved.
- **2001:** China's curriculum reform embraced Western educational ideas, emphasizing real-world contexts, inquiry-based learning, and hands-on activities, while substantially weakening traditional geometric proofs. Mathematicians and educators, including Jiang Boju and Zhang Dianzhou, publicly criticized these changes.[20, 21] In 2005, Jiang and more than 90 other experts submitted a proposal calling for revisions to the new standards.
- **2011:** Revised curriculum standards restored emphasis on systematic knowledge and logical reasoning.[22] Although transformations, practical applications, and inquiry activities remained, the Euclidean tradition of deductive proof, centered on triangle similarity, was preserved.

References:

14. Klein, Felix. *Vergleichende Betrachtungen über neuere geometrische Forschungen*. Erlangen: A. Deichert, 1872.
15. Hilbert, David. *Grundlagen der Geometrie*. Leipzig: B. G. Teubner, 1899.
16. Van Hiele, Pierre M. *Structure and Insight: A Theory of Mathematics Education*. Orlando, FL: Academic Press, 1986.
17. National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School*

Mathematics. Reston, VA: NCTM, 1989. National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

18. Klein, David, Richard Askey, R. James Milgram, Hung-Hsi Wu, Martin Scharlemann, and Betty Tsang. "An Open Letter to United States Secretary of Education, Richard Riley." *The Washington Post*, November 18, 1999.
19. National Governors Association Center for Best Practices and Council of Chief State School Officers. *Common Core State Standards for Mathematics*. Washington, DC: Authors, 2010, §§ 8.G.A.2, HSG-CO.A.2, HSG-CO.B.6.
20. Jiang, Boju. "What Has the New Curriculum Standard Taken Away from Mathematics Education?" *Guangming Daily, Education Weekly*, March 16, 2005, p. 5. (in Chinese)
21. Zhang, Dianzhou. "A Review and Prospects of Plane Geometry Teaching." *Mathematics Teaching*, 2005, No. 5: 2–7. (in Chinese)
22. Ministry of Education of the People's Republic of China. Mathematics Curriculum Standards for Compulsory Education (2011 Edition). Beijing: Beijing Normal University Press, 2012. (in Chinese)

Appendix 2: Comparison of Geometry Item Percent Correct

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This table is based on TIMSS 2019 Grade 8 mathematics item data and compares students' knowledge of geometry in the United States and East Asia. (**Percentage of students receiving full credit, %**)

Item ID	Official Item Description	United States	Singapore	Taiwan	Japan	Korea
M02_10	Determines the value of an angle in an irregular quadrilateral given the values of the other angles	**39.0**	89.7	83.4	88.8	85.9
M03_10	Finds the coordinates of a midpoint given two points in the Cartesian plane	**47.9**	58.7	65.7	72.1	61.0
M02_12	Determines the coordinates of a trapezoid's missing vertex given a congruent trapezoid in the Cartesian plane	**40.8**	66.5	73.7	77.1	67.8
M03_12	Draws rectangle on square grid given area and perimeter	**31.1**	63.6	69.5	61.1	59.8
M08_09A	Determines the area of a parallelogram given its base and height	**29.9**	64.1	74.8	74.9	71.6
M01_10	Determines the surface area of a prism given the dimensions of its net	**40.9**	73.2	67.1	68.9	67.3
M12_11	Solves for a missing side length given two similar triangles in context	**18.6**	54.5	50.1	49.2	68.4
M14_12	Justifies that a right triangle and obtuse triangle with the same base and height have the same area	**8.2**	51.1	58.5	63.6	61.3
M01_09	Identifies the image of a shape after rotation and reflection	**33.6**	35.4	43.3	54.4	31.1
M03_11	Solves for a missing side length given two similar triangles	**20.4**	53.0	35.7	29.9	74.9
M08_09B	Uses the Pythagorean theorem to solve for a side length of a parallelogram and calculates the perimeter	**13.1**	57.6	64.4	23.1	30.9

Source: TIMSS 2019 Technical Report, Appendix 15C, Grade 8 Mathematics Item Descriptions;
TIMSS 2019 International Database, Grade 8 Mathematics Item Percent Correct Statistics.

The table reveals the following patterns, offering insights into the effectiveness of different curricula:

- **Basic coordinate geometry:** U.S. students performed relatively well on basic questions involving the Cartesian coordinate system, but still lagged behind their East Asian counterparts. (M03_10 and M02_12)

- **Geometric transformations:** Rotation and reflection pose challenges for students in both East Asia and the West. (M01_09)
- **Basic geometric calculations:** Despite substantial gaps with East Asia, about one-third of American students could correctly solve basic problems involving angles, lengths, and areas. (M02_10, M03_12, M08_09A, and M01_10)
- **Complex geometric reasoning:** U.S. students performed poorly on problems requiring the application of multiple geometric properties and more than one step of reasoning. For four problems involving triangle similarity, the Pythagorean theorem, and explanations of area relationships, their percent correct were around 20% or even lower. (M12_11, M14_12, M03_11, and M08_09B)
- **Classical Euclidean proofs:** PISA and TIMSS did not directly assess classical Euclidean deductive proofs, as they have been substantially reduced in Western curricula.

(Note: some unexpectedly low scores may be due to curriculum timing, e.g., Japanese students had not yet learned the Pythagorean theorem when they took the test containing item M08_09B.)